The Rumsfeld Matrix and Weak Binding Speculations

Augusto O. Macchiavelli

Physics Division

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Nuclear Physics Over The Years: From the high spin era to rare isotopes

CELEBRATING THE CAREER AND CONTRIBUTIONS OF ROBERT V. F. JANSSENS

September 19-20, 2025

Some introductory remarks

Our ~ timeline

Ben turns 7

ATLAS Scientific Director

E. Bilchup Professor UNC

Heavy Ion Chief

ANL Physics Division Director

1988 1995 2000 2005 2010 2015 2020 2025

GAMMASPHERE Proposal

RIKEN PAC

iTHEMBA PAC

GAMMASPHERE ERA

GRETINA/ GRETA ERA

HRIBF PAC

FRIB SAC



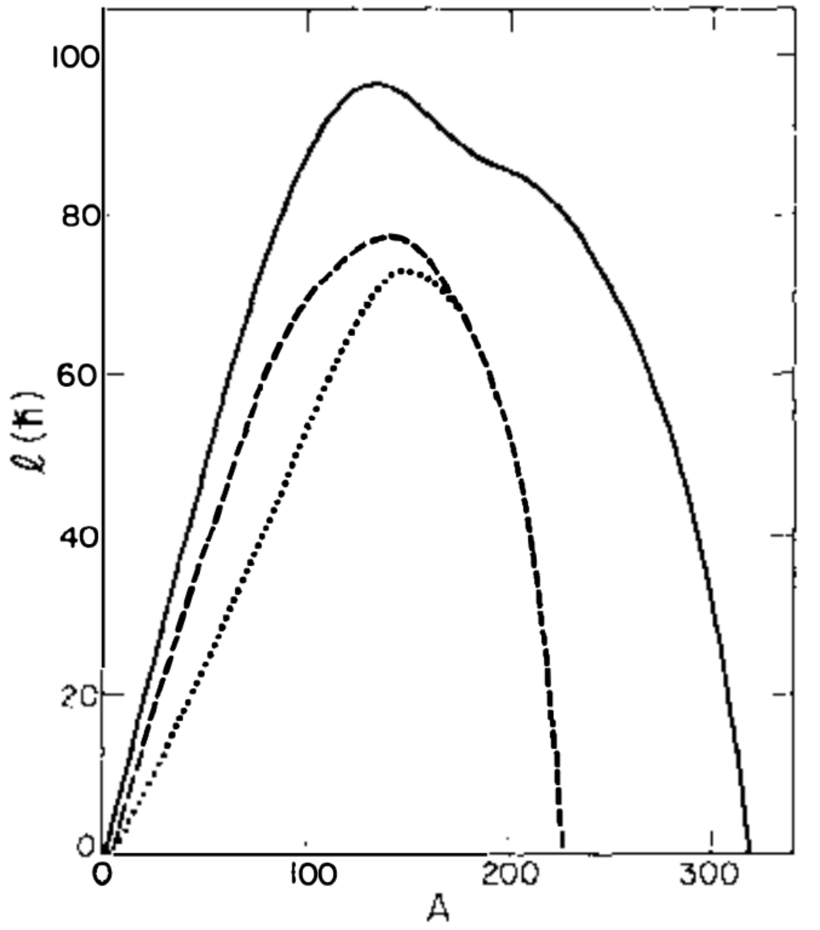

Partha's receding hearline

Last iTHEMBA PAC

The unknowns

Redefining HIGH ? SPINS

To hyphen or not to hyphen?



$$\omega = \frac{\square E}{\square I}$$ rotational frequency

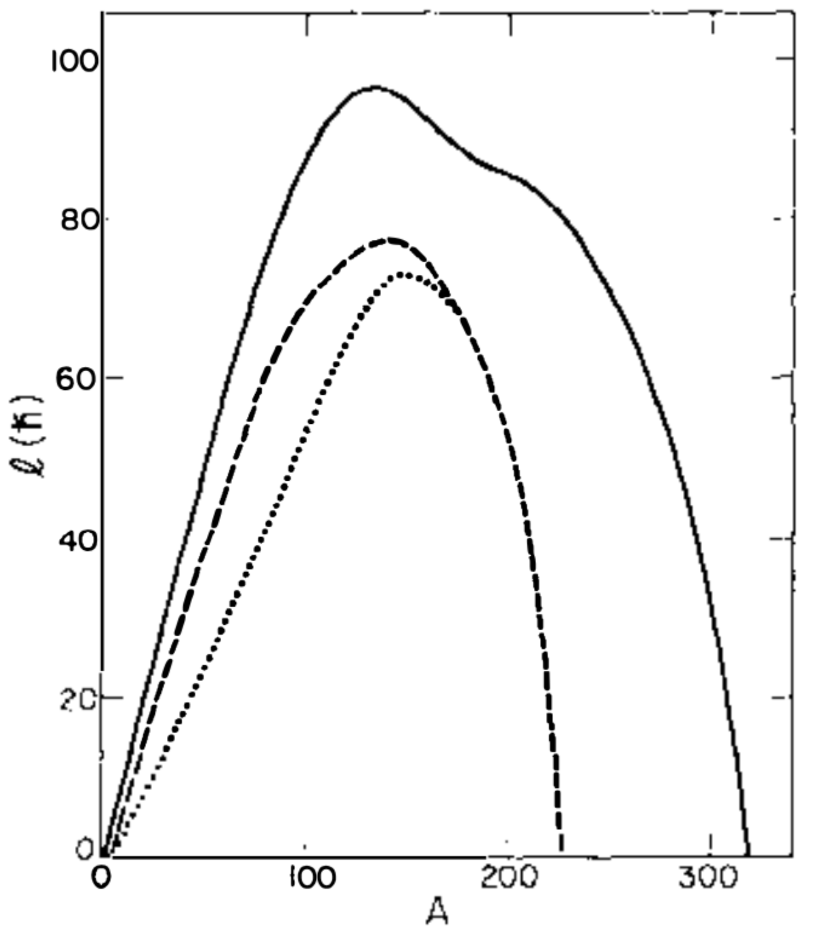
$$I(\omega)$$ angular momentum

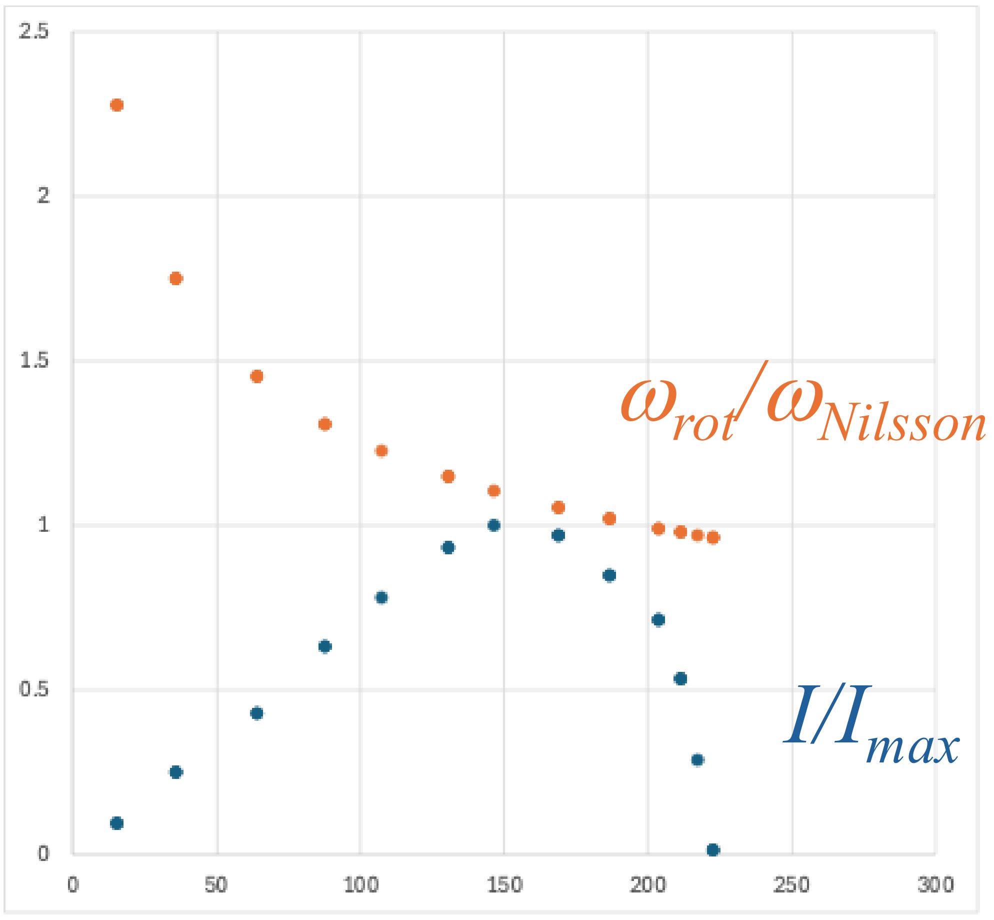
$$\square^{(1)}(\omega) = \frac{I}{\omega}$$ kinematical moment of inertia

$$\square^{(2)}(\omega) = \frac{dI}{d\omega}$$ dynamical moment of inertia

$$\omega_{Nilsson} \sim \varepsilon\, \omega_0$$

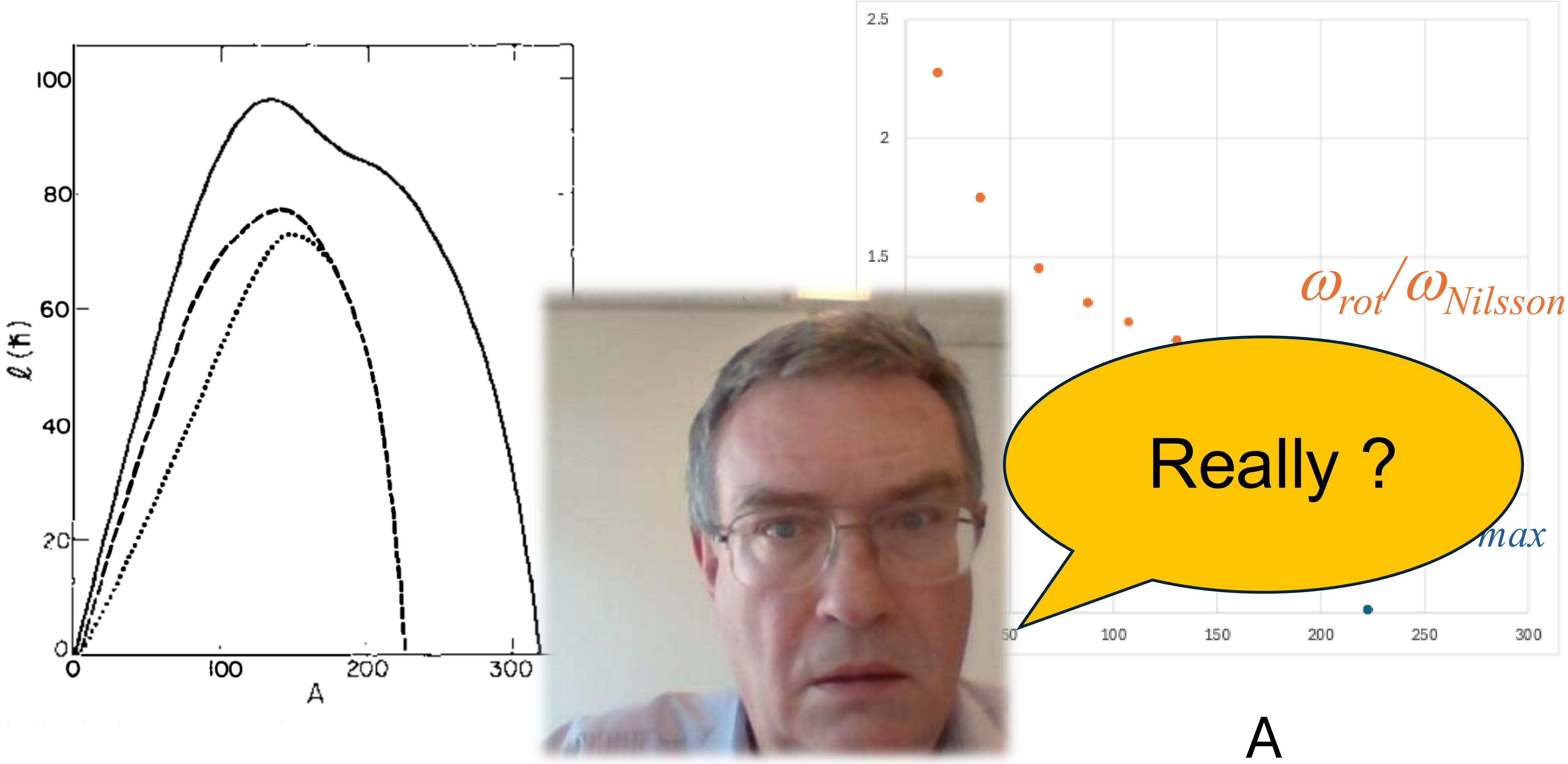
Redefining HIGH?SPINS





A

Redefining HIGH?SPINS

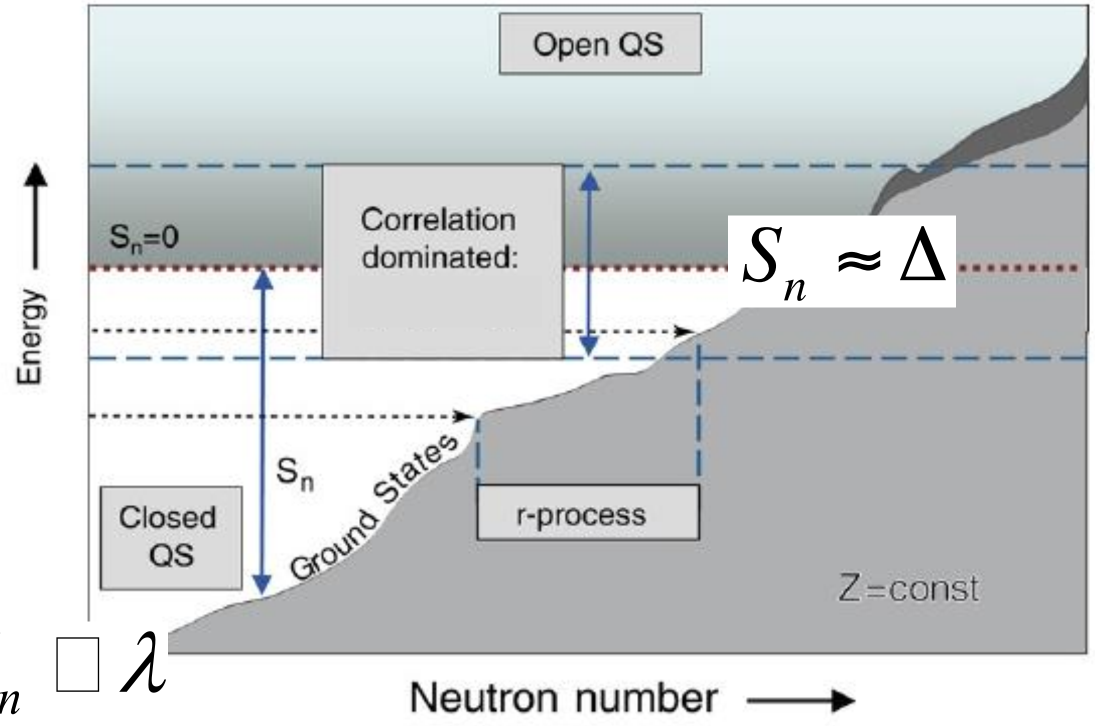


The Rumsfeld Matrix

	Knowns	Unknowns
Knowns	**Known Knowns** Things we are aware of and understand.	**Known Unknowns** Things we are aware of but don't understand.
Unknowns	**Unknown Knowns** Things we understand but are not aware of.	**Unknown Unknowns** Things we are neither aware of nor understand.

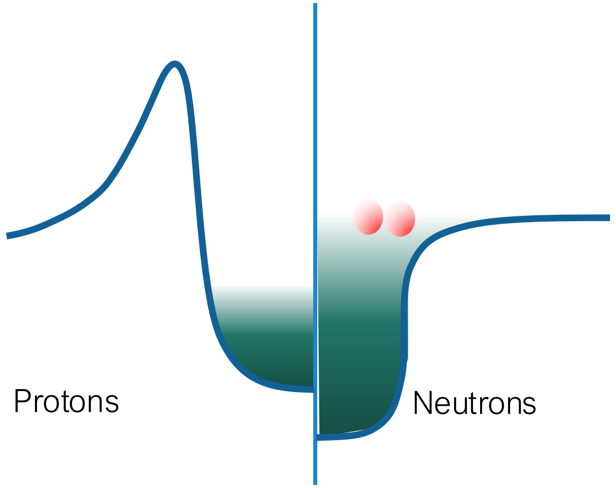
Weakly bound systems

J. Dobaczewski et al. / Progress in Particle and Nuclear Physics 59 (2007) 432–445



$$S_n \approx \Delta$$

$$S_n \square \lambda$$



$$\lambda / \Delta \approx 1$$

$$S_n \approx \Delta + \lambda$$

- Coupling to the Continuum

Near the valley of stability, the quasi-particle binding is dominated by the mean field and $S_n \approx \lambda$. As we increase the number of neutrons, approaching the drip line, $\lambda \to 0$ and $S_n \approx \Delta$, ie. correlations dominated.

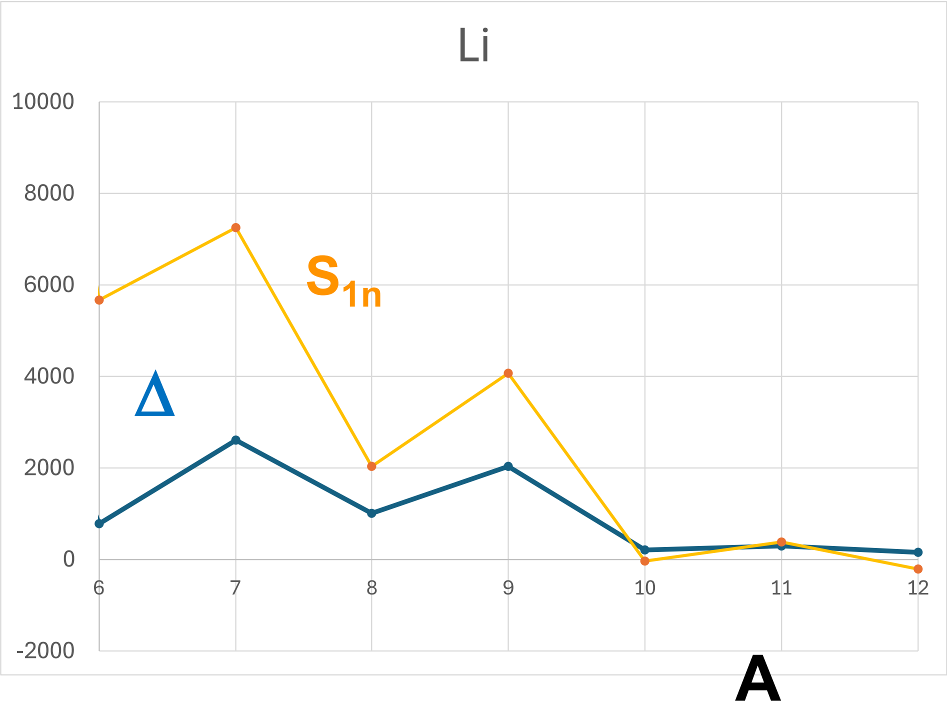
Thus, it is natural to expect that the transition between the two regimes will start to take place when $S_n \approx \Delta$.

The above can also be related to the asymptotic behavior of the Cooper pairs $\to e^{-Kr}$.

For strongly bound nuclei, $K \approx 2\kappa$, the tail of the particle density.

$$K \approx 2\kappa = 2\sqrt{2mS_{1n}}/\hbar$$

For weakly bound nuclei, $K < 2\kappa$ and the pair extends further outside the surface.



I. Tanihata et al. / Progress in Particle and Nuclear Physics 68 (2013) 215–313

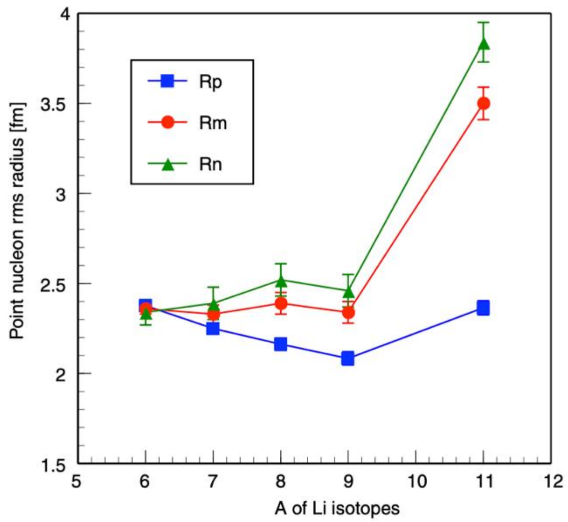


Fig. 3.10. Radii of point nucleon distribution in Li isotopes.

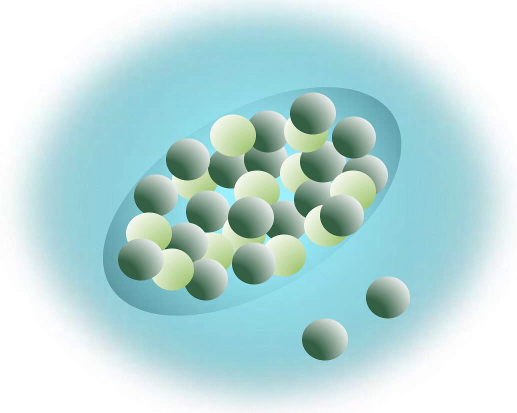
Motivation I

Rotational Motion

Islands of Inversion → deformation

Weak binding

Neutron-rich nuclei → drip-line



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PHYSICAL REVIEW LETTERS **122,** 052501 (2019)

Editors' Suggestion

First Spectroscopy of the Near Drip-line Nucleus ^{40}Mg

H. L. Crawford,[1,*] P. Fallon,[1] A. O. Macchiavelli,[1] P. Doornenbal,[2] N. Aoi,[3] F. Browne,[2] C. M. Campbell,[1] S. Chen,[2] R. M. Clark,[1] M. L. Cortés,[2] M. Cromaz,[1] E. Ideguchi,[3] M. D. Jones,[1,†] R. Kanungo,[4,5] M. MacCormick,[6] S. Momiyama,[7] I. Murray,[6] M. Niikura,[7] S. Paschalis,[8] M. Petri,[8] H. Sakurai,[2,7] M. Salathe,[1] P. Schrock,[9] D. Steppenbeck,[9] S. Takeuchi,[2,10] Y. K. Tanaka,[11] R. Taniuchi,[7] H. Wang,[2] and K. Wimmer[7]

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https://doi.org/10.1140/epja/s10050-022-00719-5

THE EUROPEAN PHYSICAL JOURNAL A

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Regular Article - Theoretical Physics

Weak binding effects on the structure of ^{40}Mg

A. O. Macchiavelli[1,a], H. L. Crawford[1], P. Fallon[1], R. M. Clark[1], A. Poves[2]

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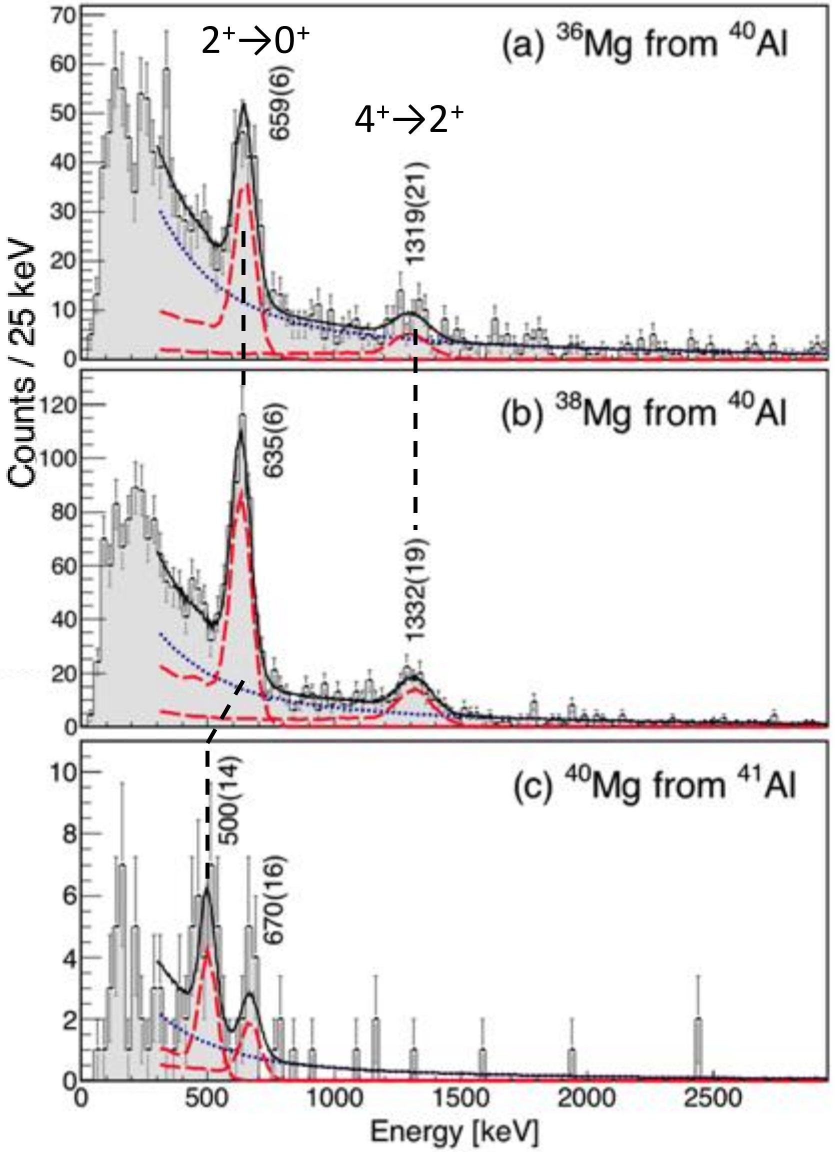
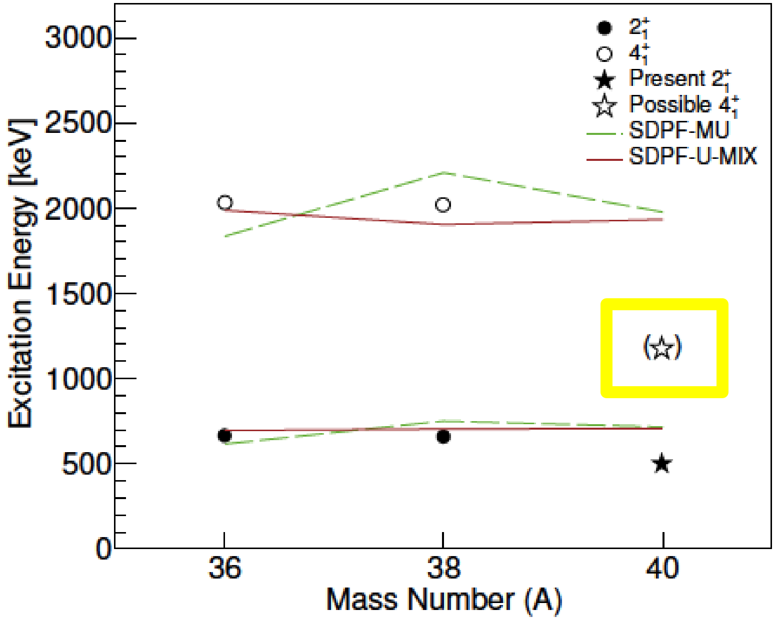
[2] Departamento de Física Teórica and IFT-UAM/CSIC, Universidad Autónoma de Madrid, 28049 Madrid, Spain

Results: ^{40}Mg (670 keV transition)





- **670 keV transition ?**

4_1^+ □ 2_1^+ | 2_2^+ □ 0_1^+ * | 2_2^+ □ 2_1^+ | 0_2^+ □ 2_1^+ + ...

- No scenario fits with existing expectations (systematics) nor predictions from calculation
- Breakdown of systematics and theory predictions may suggest something is happening at the dripline ??

* **preferred, cf.** Crawford *et al.*, PRC 89, 041303(R) (2014).

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P.G. Hansen, B. Jonson, Europhys. Lett. 4, 409 (1987)

As discussed by Hansen and Jonson and subsequent works the extended matter radius exhibited by a two-neutron halo nucleus can be expressed in terms of the separation energy of the weakly bound neutrons (S_{2n}) through the tunneling parameter

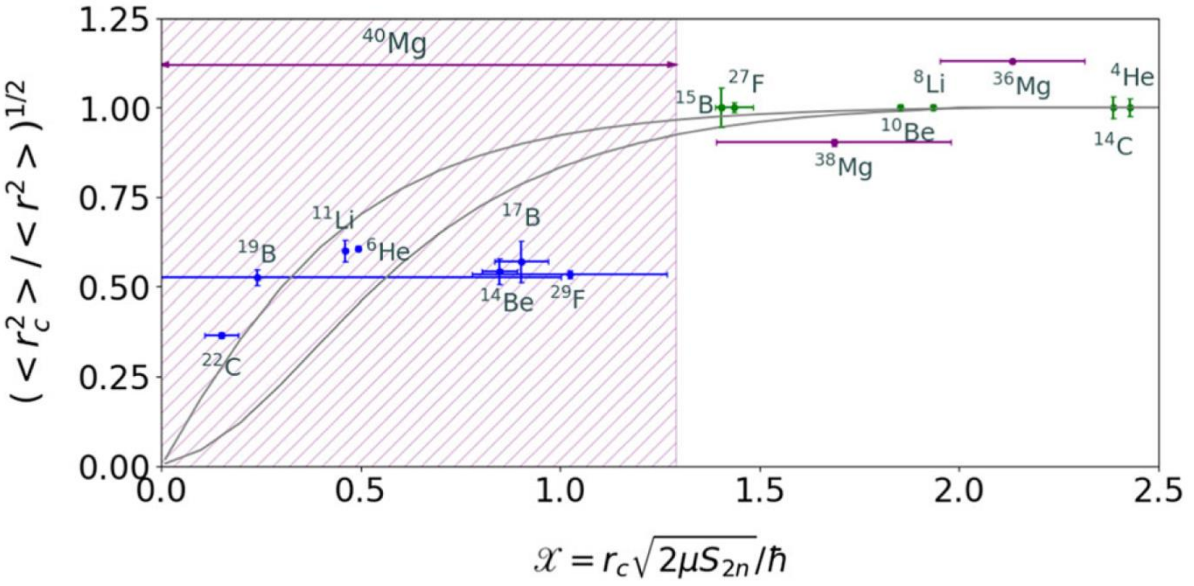
$$\mathscr{X} = r_c\sqrt{2\mu S_{2n}}/\hbar$$

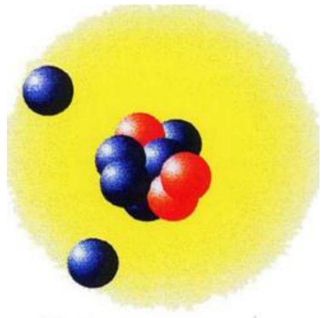
derived from the exponential nature of the asymptotic wavefunction. Following these authors, we consider a plot of

$$(< r_c^2 > / < r_m^2 >)^{1/2} \text{ vs. } \mathscr{X}$$

to capture the universal features of the *2n* halo systems. Their root-mean-square (RMS) ratio represents the volume overlap between the valence nucleons and the core.

S_{2n} of 667±706 keV.





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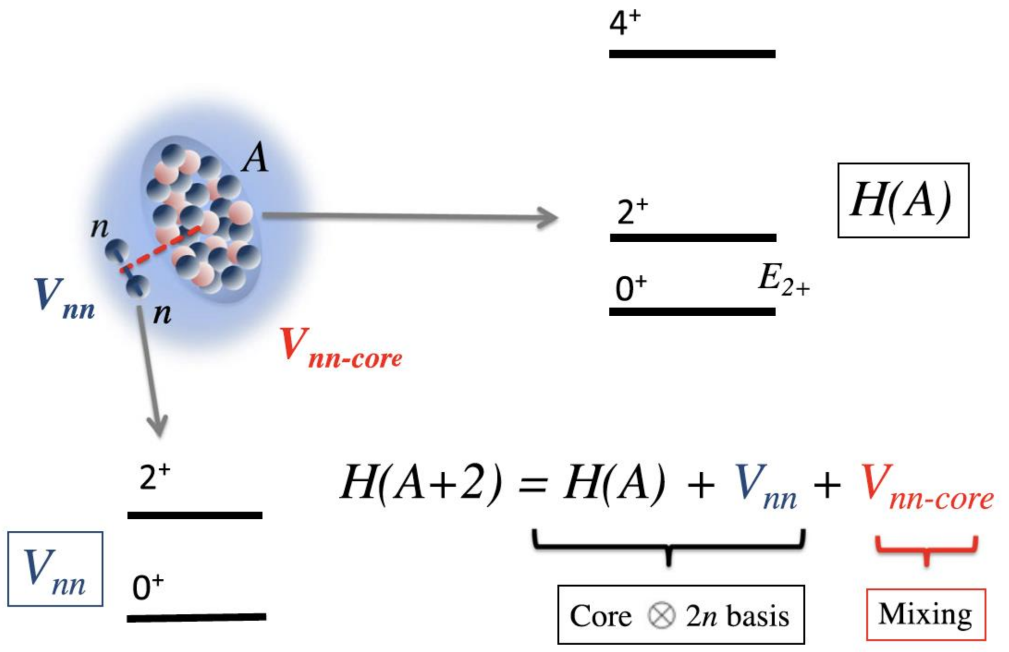
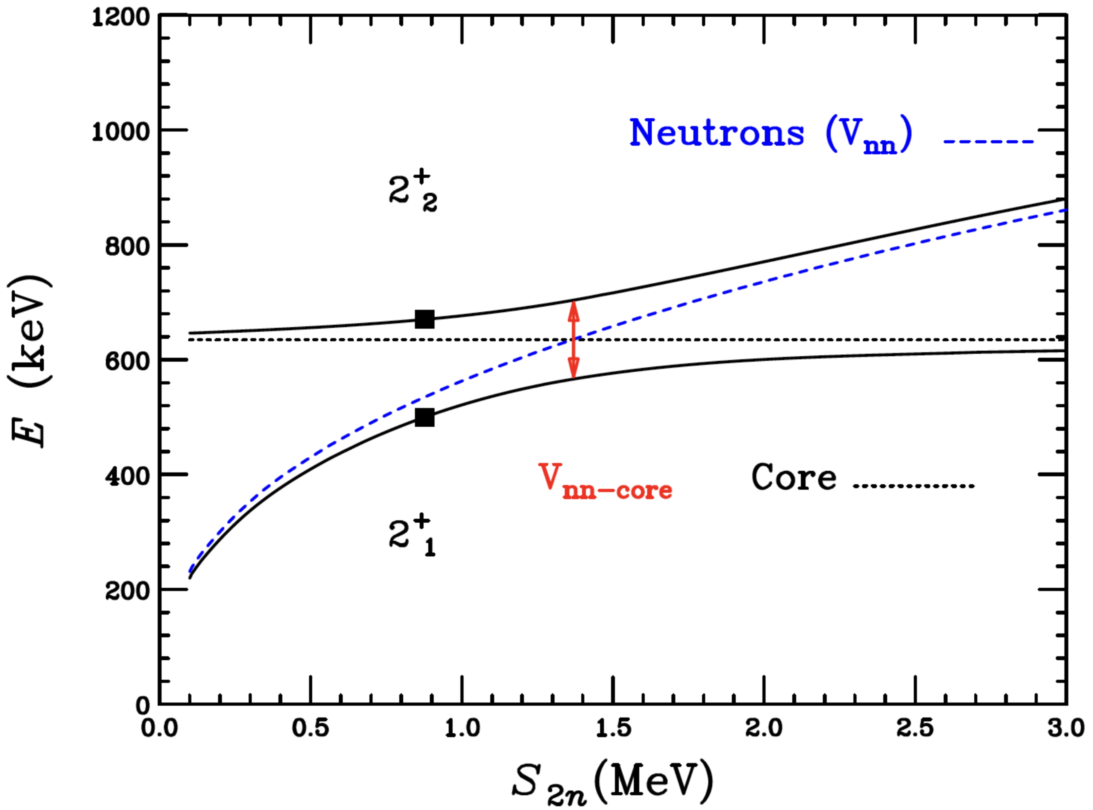
Weak coupling phenomenological model



It is natural to expect that effects of weak binding on excited states will show when the energy scales of the two degrees of freedom become comparable:

$$E_{core}(2^+) \approx E_{2n}(2^+)$$

State energies and wavefunctions



It is interesting to see that the unperturbed lines cross for binding energies in the range expected for ^{40}Mg and even a small mixing matrix element $Vnn{-}core$ will give rise to largely mixed states in the laboratory frame.

A minimization procedure on the experimental energies of the two potential states populated gives a solution with

$Vnn{-}core$ = 69keV $\quad$ S_{2n} = 877 keV

and wavefunctions:

$$|2_1^+\rangle = 0.45|2^+_{core}\rangle + 0.89|2^+_{2n}\rangle$$
$$|2_2^+\rangle = -0.89|2^+_{core}\rangle + 0.45|2^+_{2n}\rangle$$

The fact that $Vnn{-}core$ << E_{core} (V_{nn}) supports the weak coupling assumption.

Reaction cross sections

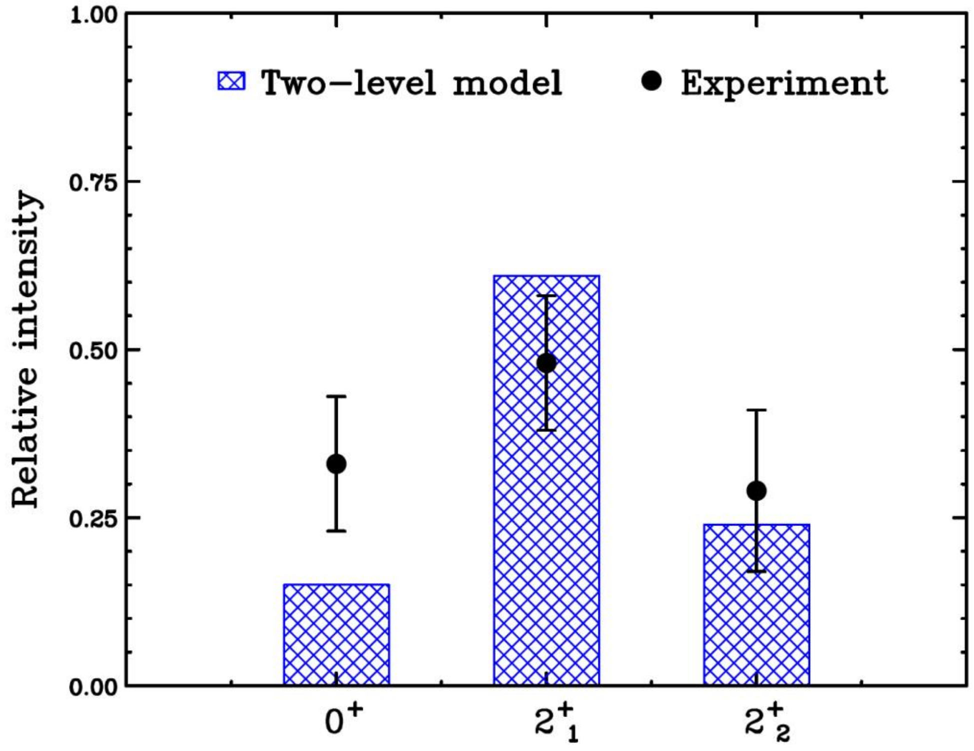
The wavefunctions of the two 2^+ states can readily be used to determine their relative intensities populated in a direct knockout reaction

To calculate the population of the final states in ^{40}Mg produced from the ^{41}Al(-1p) reaction we assume that the ground state of ^{41}Al is $K = 5/2^+$, from the π[202]5/2 Nilsson level originating from the $d_{5/2}$ spherical level.

In the single-j approximation the collective spectroscopic factors follow the values of the Clebsch-Gordan coefficients:

$$\langle \tfrac{5}{2}\tfrac{5}{2}\tfrac{5}{2}-\tfrac{5}{2}|I_f 0\rangle$$

In the minimization procedure we also include a single-particle spectroscopic factor $S_{sp}(5/2+ \rightarrow 2^+_{2n})$ with a fitted value of 0.14. This gives a measure of the component of the $|2^+_{2n}>$ state in the ground state of ^{41}Al.



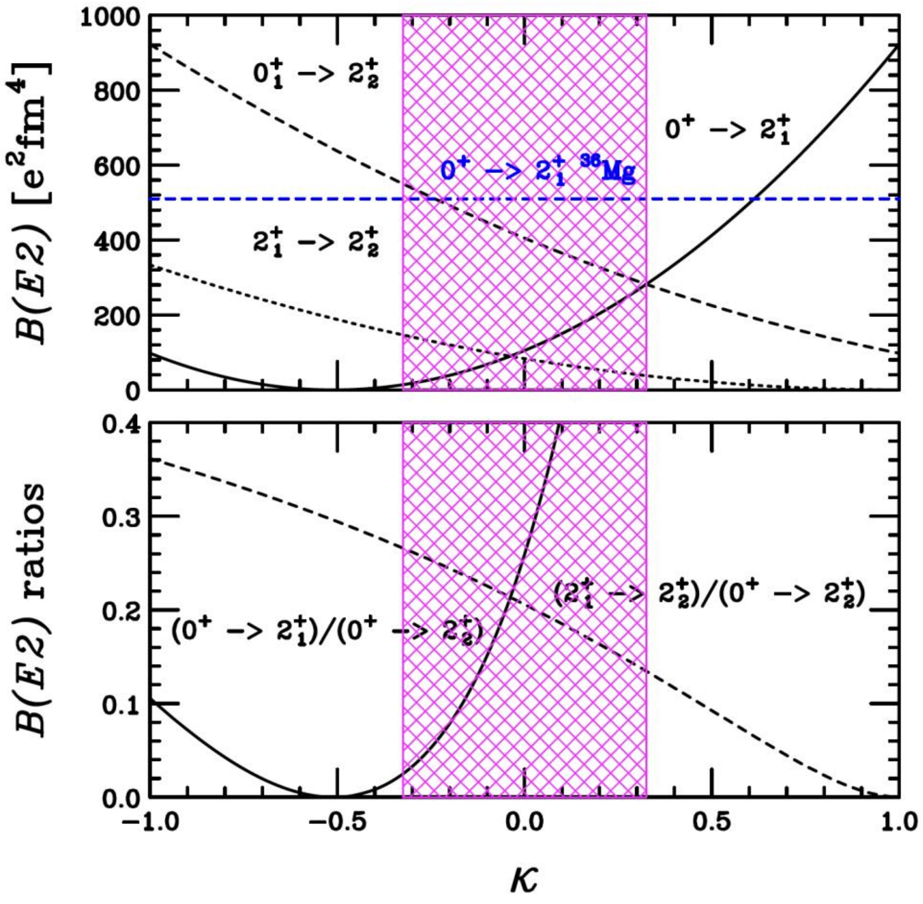
Transition probabilities

It is of interest to consider E2 transition probabilities between the low-lying states, in particular the contribution of the neutron halo to excite the 2^+_1 and 2^+_2 states, for example in an intermediate energy Coulomb excitation experiment

In the weak coupling framework, we can calculate the transition probabilities :

$$B(E2, 0^+ \rightarrow 2_1^+) \approx (\alpha + \beta\kappa)^2 B(E2)_{core}$$
$$B(E2, 0^+ \rightarrow 2_2^+) \approx (\beta - \alpha\kappa)^2 B(E2)_{core}$$
$$B(E2, 2_1^+ \rightarrow 2_2^+) \approx (\alpha\beta(1 - \kappa))^2 B(E2)_{core}$$

$$\kappa = \langle 0^+|\mathcal{M}(E2)_{2n}|2^+_{2n}\rangle / \langle 0^+|\mathcal{M}(E2)_{core}|2^+_{core}\rangle =$$
$$\langle 2^+_{2n}|\mathcal{M}(E2)_{2n}|2^+_{2n}\rangle / \langle 2^+_{core}|\mathcal{M}(E2)_{core}|2^+_{core}\rangle$$



$$g(2_1^+) = \alpha^2 \frac{Z}{A} + \beta^2 \frac{1}{2} g_{s,\nu}$$

giving $g(2_1^+) \approx -1.45\ \mu_N$.

PHYSICAL REVIEW LETTERS **129,** 212502 (2022)

Editors' Suggestion | Featured in Physics

Discovery of ^{39}Na

D. S. Ahn,[1,*] J. Amano,[3] H. Baba,[1] N. Fukuda,[1] H. Geissel,[5] N. Inabe,[1] S. Ishikawa,[4] N. Iwasa,[4] T. Komatsubara,[1] T. Kubo,[1,†] K. Kusaka,[1] D. J. Morrissey,[6] T. Nakamura,[2] M. Ohtake,[1] H. Otsu,[1] T. Sakakibara,[4] H. Sato,[1] B. M. Sherrill,[6] Y. Shimizu,[1] T. Sumikama,[1] H. Suzuki,[1] H. Takeda,[1] O. B. Tarasov,[6] H. Ueno,[1] Y. Yanagisawa,[1] and K. Yoshida[1]

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[6]*National Superconducting Cyclotron Laboratory, Michigan State University, 640 South Shaw Lane, East Lansing, Michigan 48824, USA*

The new isotope ^{39}Na, the most neutron-rich sodium nucleus observed so far, was discovered at the RIKEN Nishina Center Radioactive Isotope Beam Factory using the projectile fragmentation of an intense ^{48}Ca beam at 345 MeV/nucleon on a beryllium target. Projectile fragments were separated and identified in flight with the large-acceptance two-stage separator BigRIPS. Nine ^{39}Na events have been unambiguously observed in this work and clearly establish the particle stability of ^{39}Na. Furthermore, the lack of observation of ^{35,36}Ne isotopes in this experiment significantly improves the overall confidence that ^{34}Ne is the neutron dripline nucleus of neon. These results provide new key information to understand nuclear binding and nuclear structure under extremely neutron-rich conditions. The newly established stability of ^{39}Na has a significant impact on nuclear models and theories predicting the neutron dripline and also provides a key to understanding the nuclear shell property of ^{39}Na at the neutron number $N = 28$, which is normally a magic number.

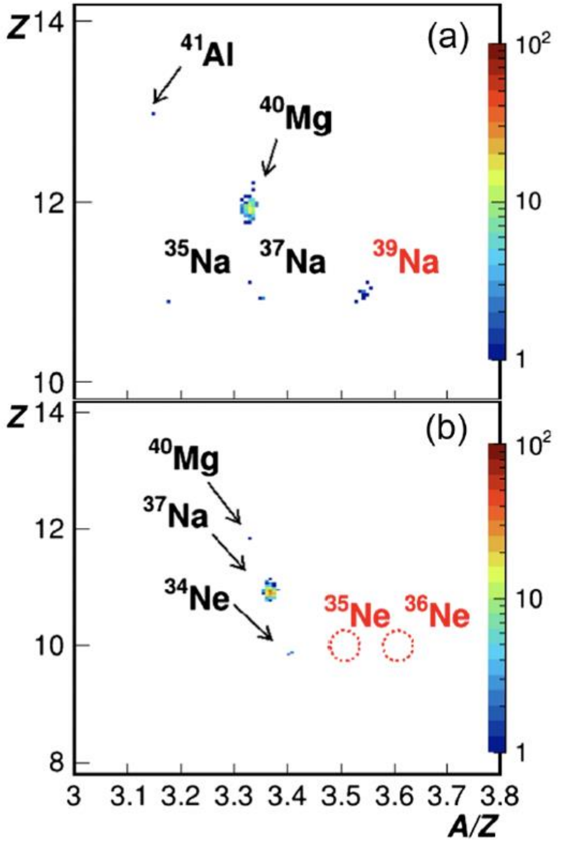


FIG. 1. Z versus A/Z particle identification plots for projectile fragments produced in the ^{48}Ca + Be reaction at 345 MeV/nucleon are shown for the (a) ^{39}Na and (b) ^{36}Ne settings. Nine events were observed for ^{39}Na in the ^{39}Na setting. No events were observed for ^{35,36}Ne in the ^{36}Ne setting.

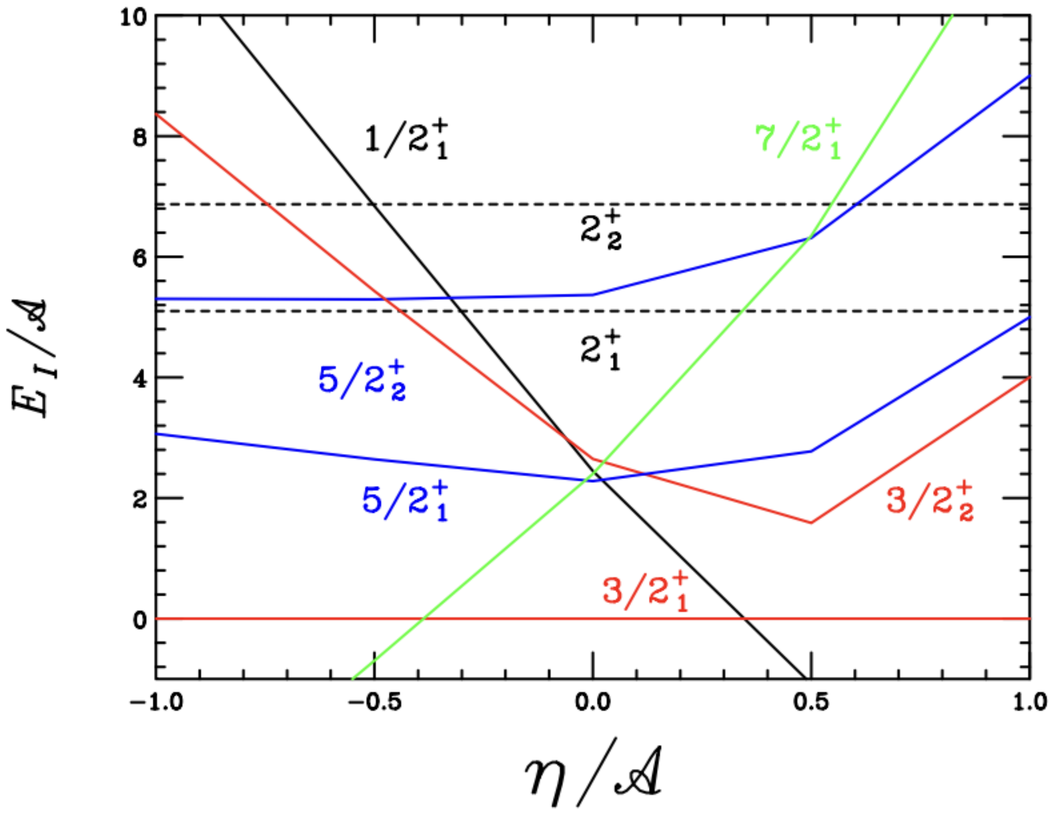
^{39}Na low-lying structure

Consider for example the case of ^{39}Na with a single proton in the [211]3/2 Nilsson orbit, a proton hole outside of ^{40}Mg.

$$\eta \vec{j}_{2n} \cdot \vec{j}_p$$

Residual np interaction

With an estimate of the residual np interaction of η ≈ 34/A MeV, derived from the analysis of pairing gaps , we anticipate a value of |η| <~ 0.1 MeV for A = 40, depending on the volume overlap correction.



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Motivation II: Superradiance

Superradiance was first studied by Dicke within the context of coherence effects in spontaneous radiation processes. Since then, the phenomenon has been referenced in many areas of modern science, among them: quantum optics, condensed matter, biophysics, and nuclear physics.

In atomic nuclei, seen as a complex open quantum many-body system, the effect arises from the coupling to continuum states that can be treated in terms of a non-hermitian hamiltonian (non- hermitian super-radiance). Increasing coupling to the continuum leads to the separation of long- lived and short-lived (superradiant) resonance states.

R. J. Dicke, Phys. Rev. 93, 99 (1954)

P. von Brentano, Physics Report 264, 57 (1996) 57
A. Volya, V. Zelevinsky, AIP Conf.Proc. 777, 229 (2004)
N. Auerbach, V. Zelevinsky, Rep. Prog. Phys. 74, 106301 (2011)
I. Rotter, J.P. Bird, Rep. Prog. Phys. 78, 114001 (2015)

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physics

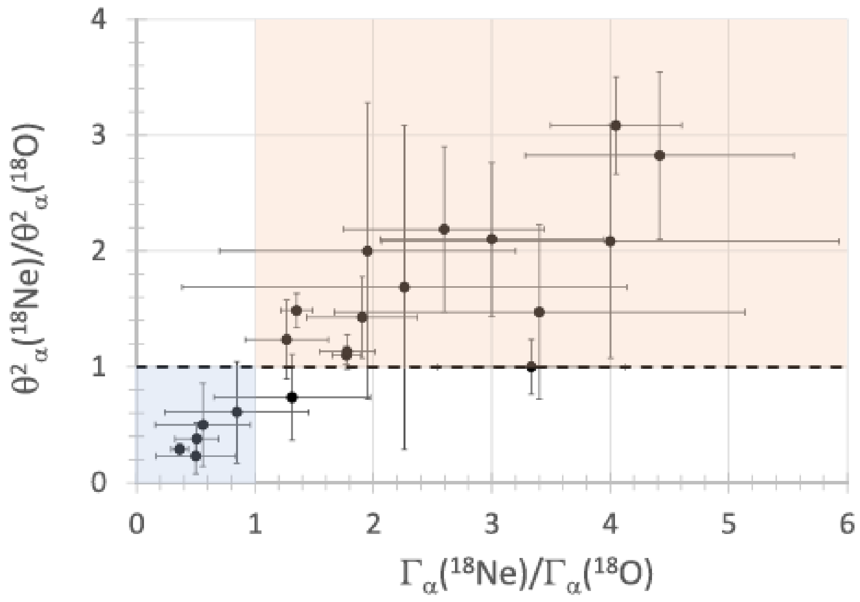
ARTICLE

https://doi.org/10.1038/s42005-022-01105-9 OPEN

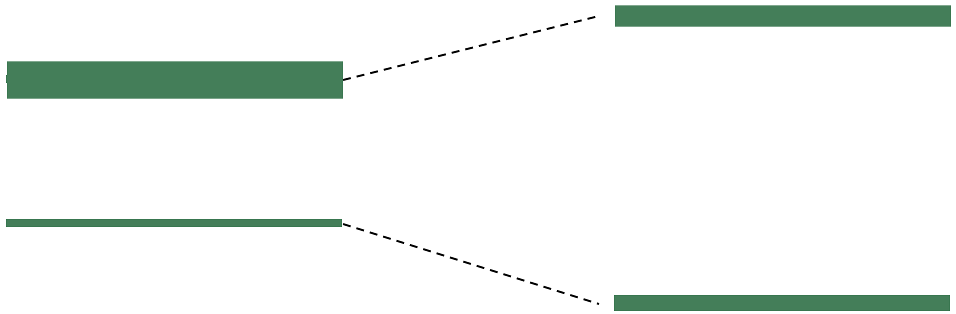
Superradiance in alpha clustered mirror nuclei

Alexander Volya [1,2], Marina Barbui [2], Vladilen Z. Goldberg[2] & Grigory V. Rogachev[2,3,4]

includes other decay channels are necessary. It would also be interesting to use different reactions, such as alpha-transfer, to populate the cluster states and provide an independent measure of the total width and branching ratios in mirror nuclei to verify and benchmark current findings. Yet, our findings here may be the clearest manifestation of the superradiance phenomenon in nuclear physics to date.



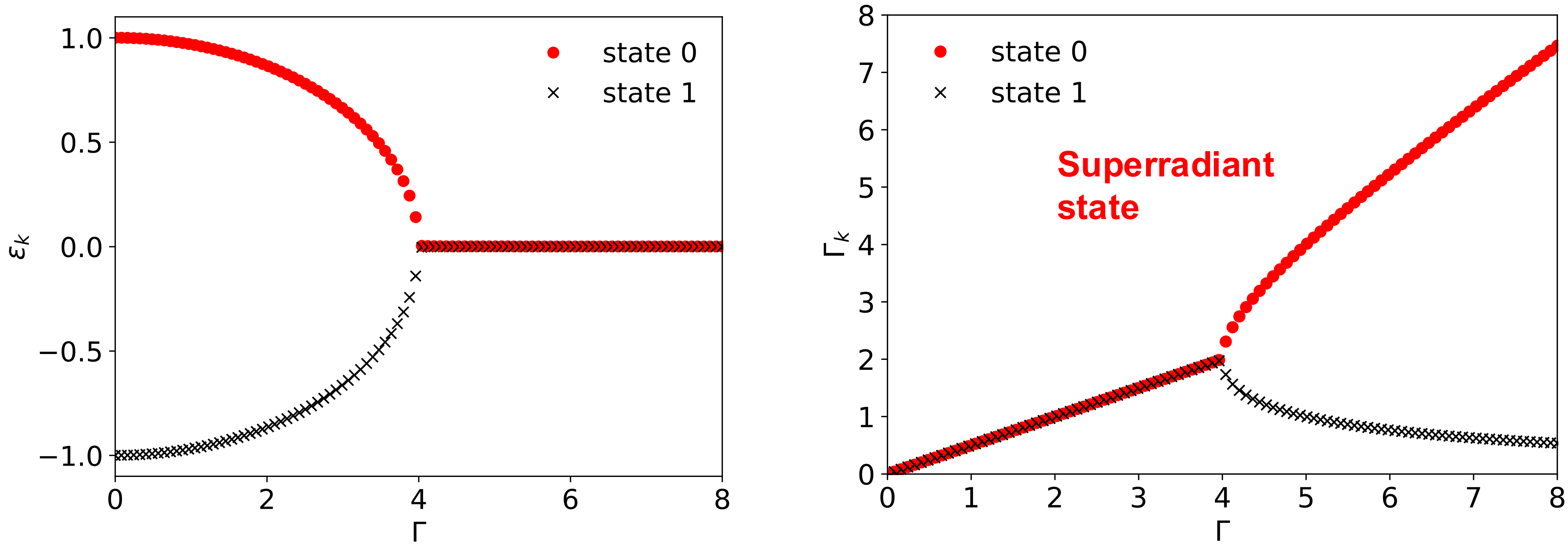
What is superradiance anyway?



Mixing of unbound levels

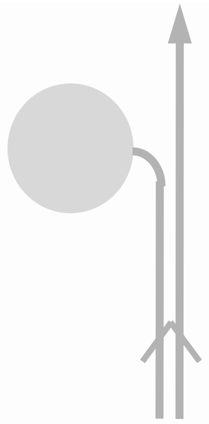
$$H = \begin{pmatrix} \epsilon - \frac{i}{2}\Gamma & v \\ v & 0 \end{pmatrix}$$

$$\mathcal{E}_{1,2} = \frac{1}{2}\left(\epsilon - \frac{i}{2}\Gamma \pm \sqrt{\left(\epsilon - \frac{i}{2}\Gamma\right)^2 + 4v^2}\right)$$



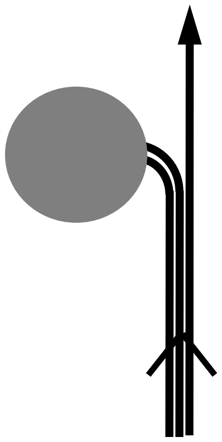
Simplest case degenerate levels, $\varepsilon = 0$

Direct Reactions



$$< A+1 \mid a^{+} \mid A >$$

Spectroscopic (*u, v*) Factors



$$< A+2 \mid a^{+}a^{+} \mid A >$$

Constructive interference

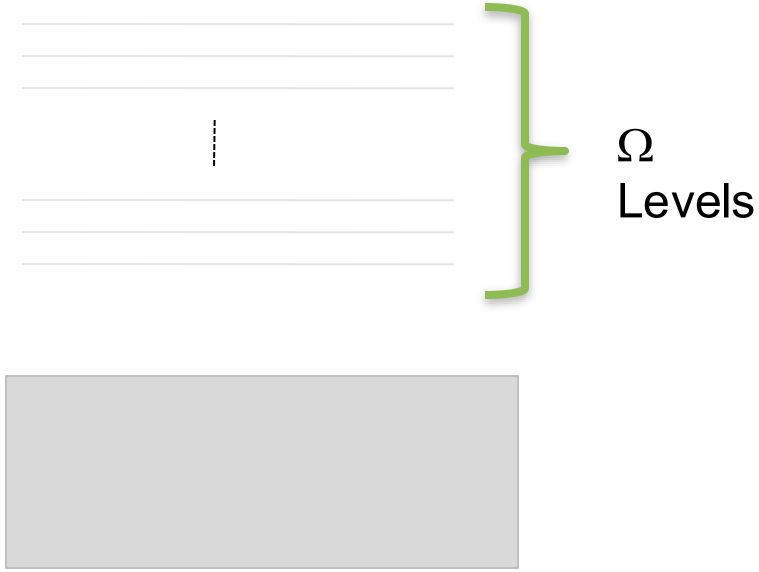
Two particle transfer reactions like (t,p) or (p,t), where 2 nucleons are deposited or picked up at the same point in space provide a specific tool to probe the amplitude of this collective motion.

The transition operators $<f|a^{+}a^{+}|i>$, $<f|aa|i>$ are the analogous to the transition probabilities *B(E2)*'s on the quadrupole case.

R.A. Broglia, O. Hansen and C. Riedel, Adv. Nucl. Phys. Vol 6 (1973) 287

D. M. Brink and R.A. Broglia, Nuclear Superfluidity, Cambridge Monographs.

Pairing vibrations and the (t,p) reaction



Closed shell nucleus, A_0

$$\sigma \propto < A_o + 2 | T | A_o >^2$$

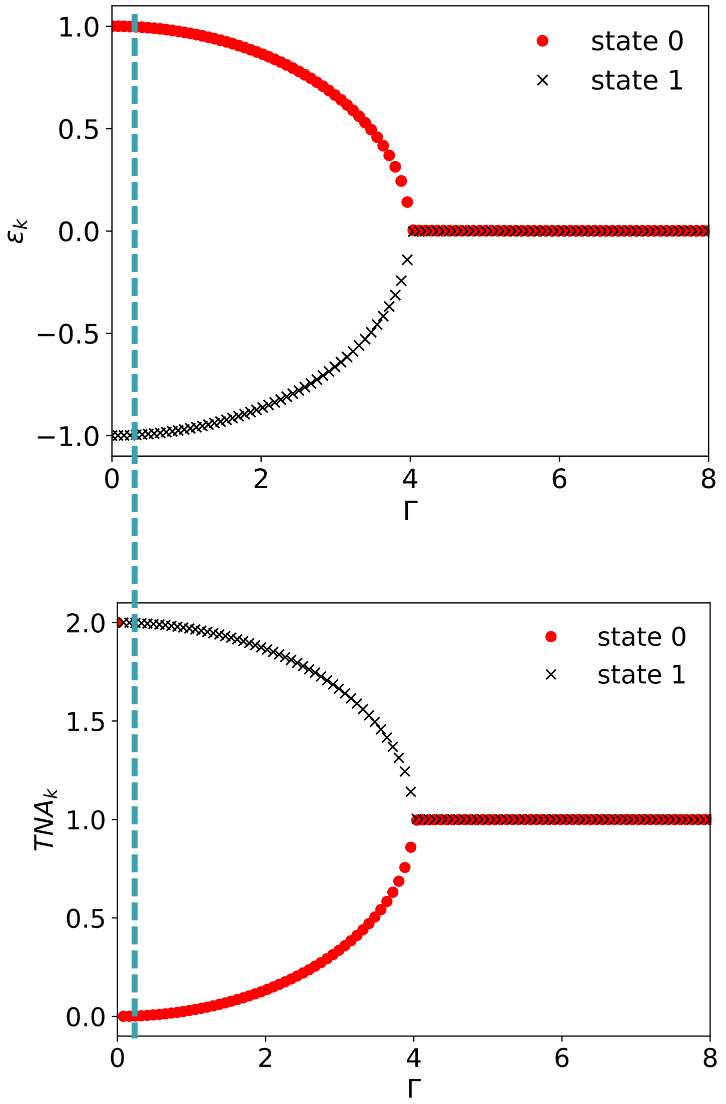
$$| A_o + 2 > = \sum_i \frac{1}{\sqrt{\Omega}} | i >$$

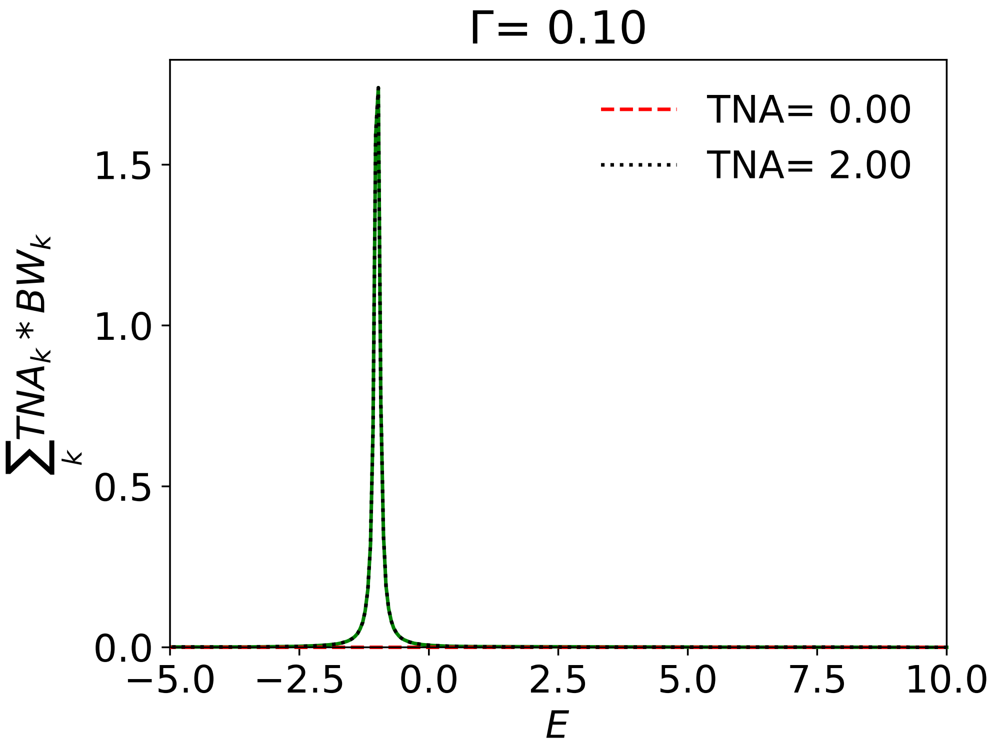
$$\sigma \propto \left(\sum_i \frac{1}{\sqrt{\Omega}} < i | T | o > \right)^2$$

$$\sigma \approx \Omega \sigma_{sp}$$

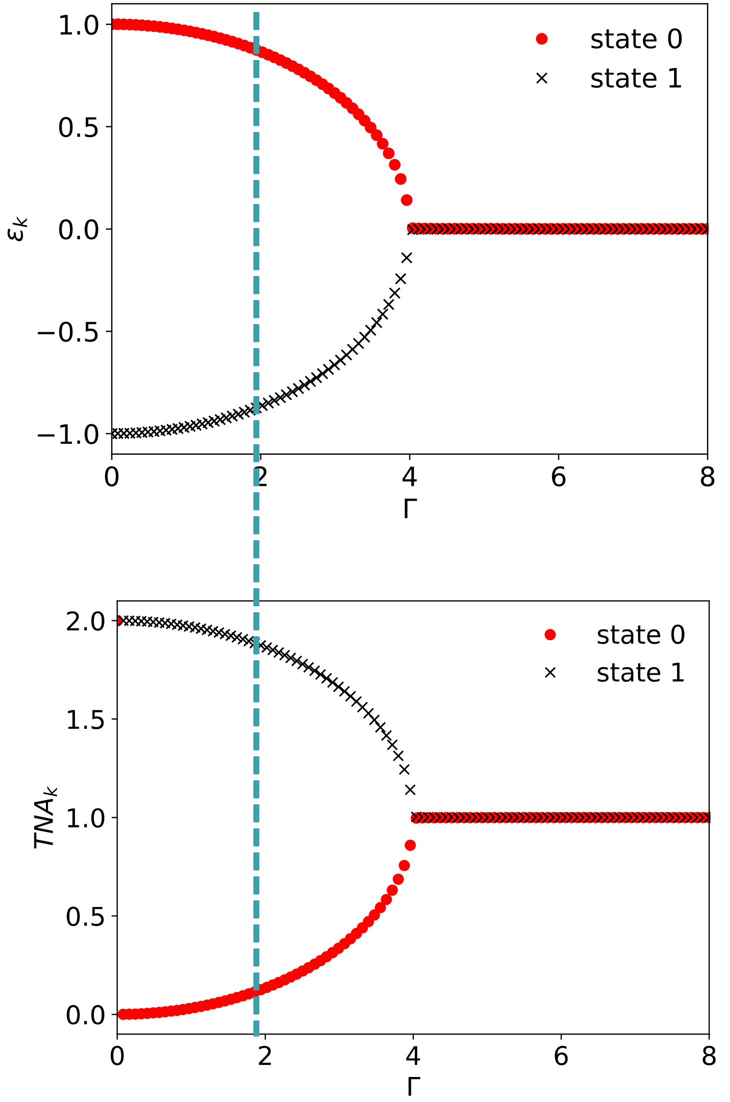
Collective enhancement over *sp* cross-section due to coherent contributions of correlated *nn* pairs

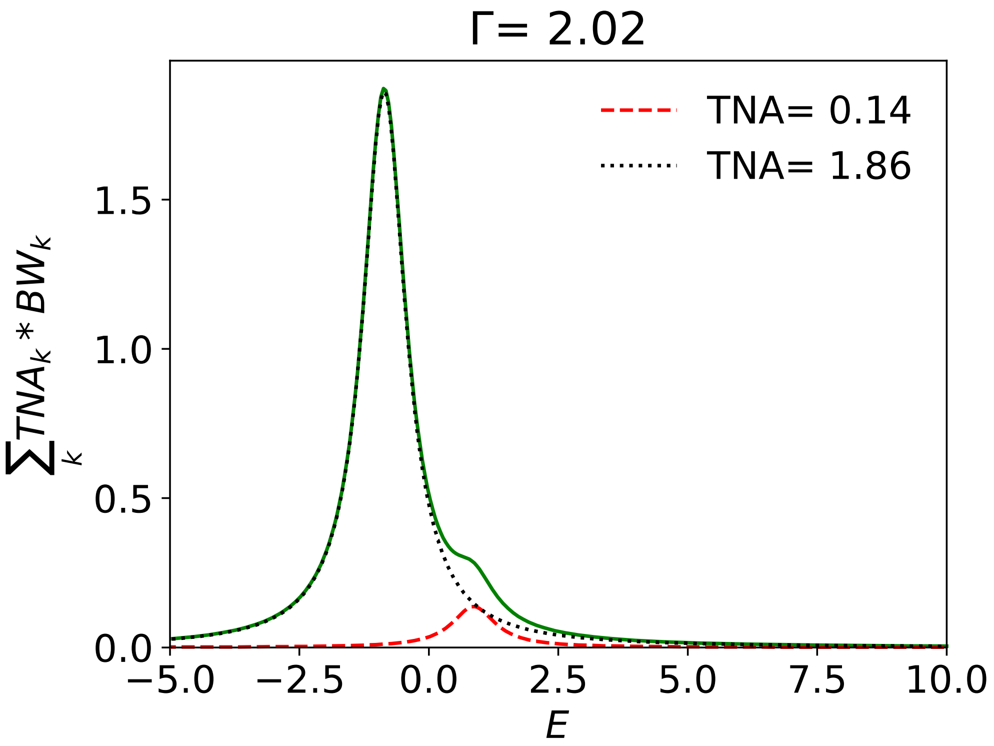
Superradiance and two-neutron transfer reactions



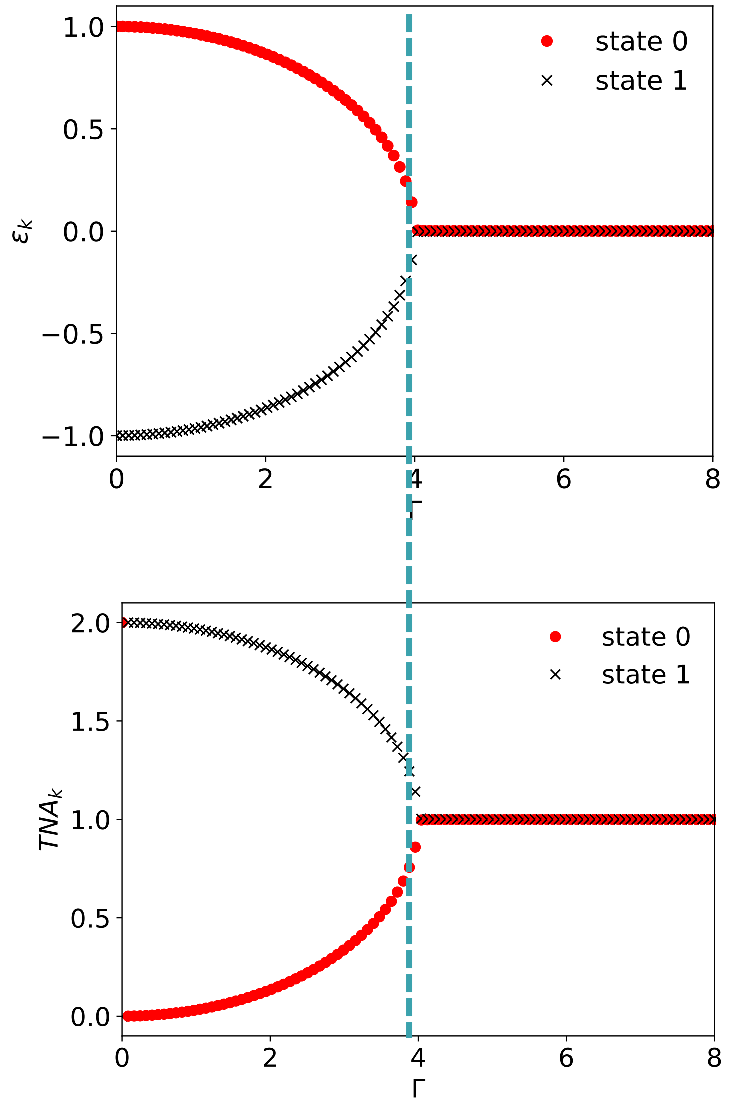


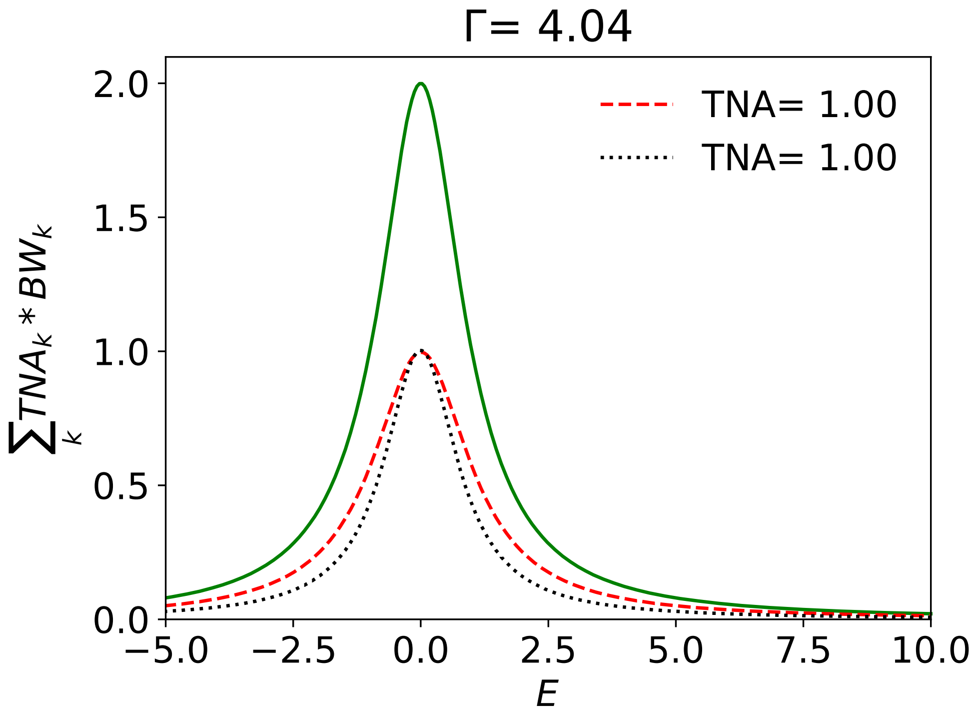
Superradiance and two-neutron transfer reactions



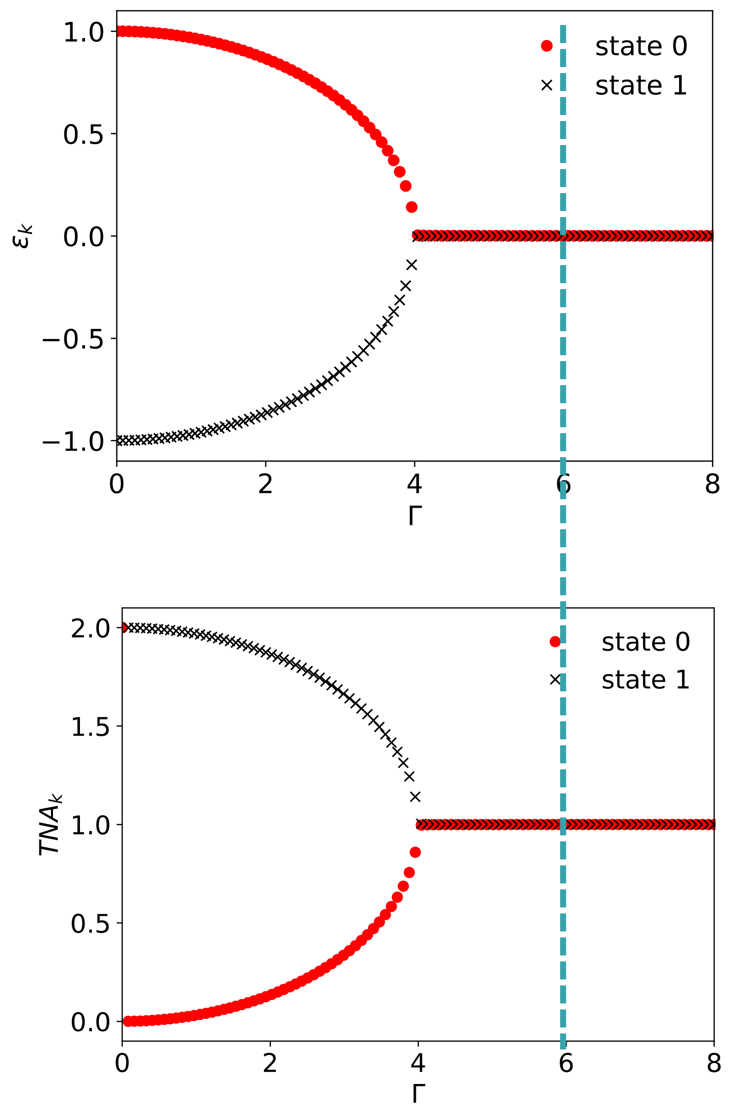


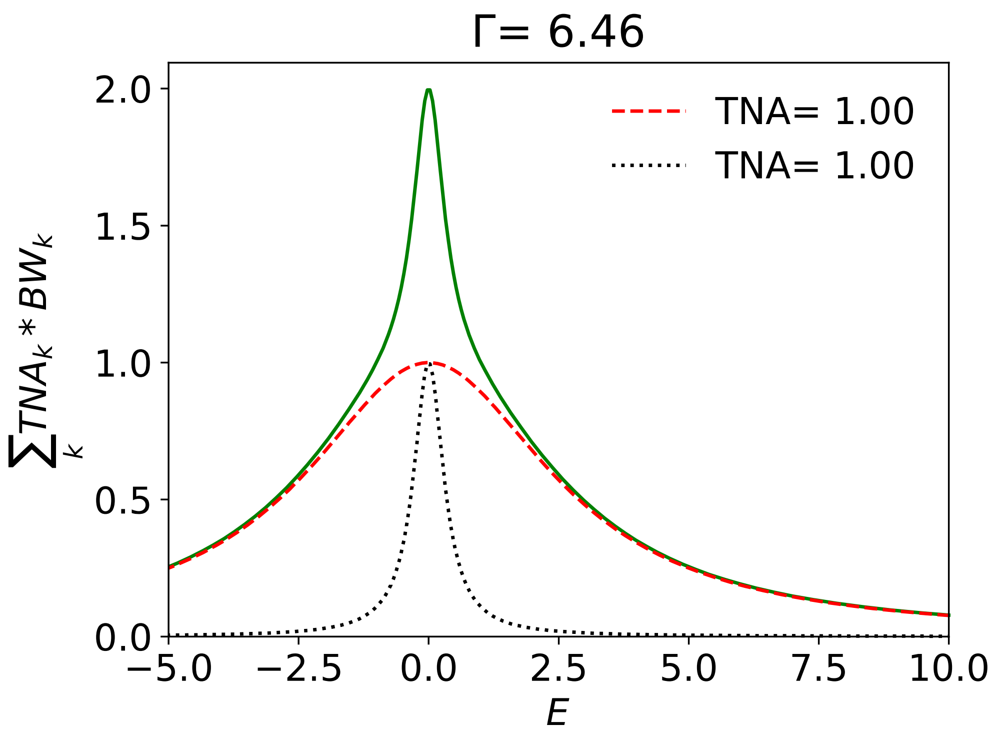
Superradiance and two-neutron transfer reactions



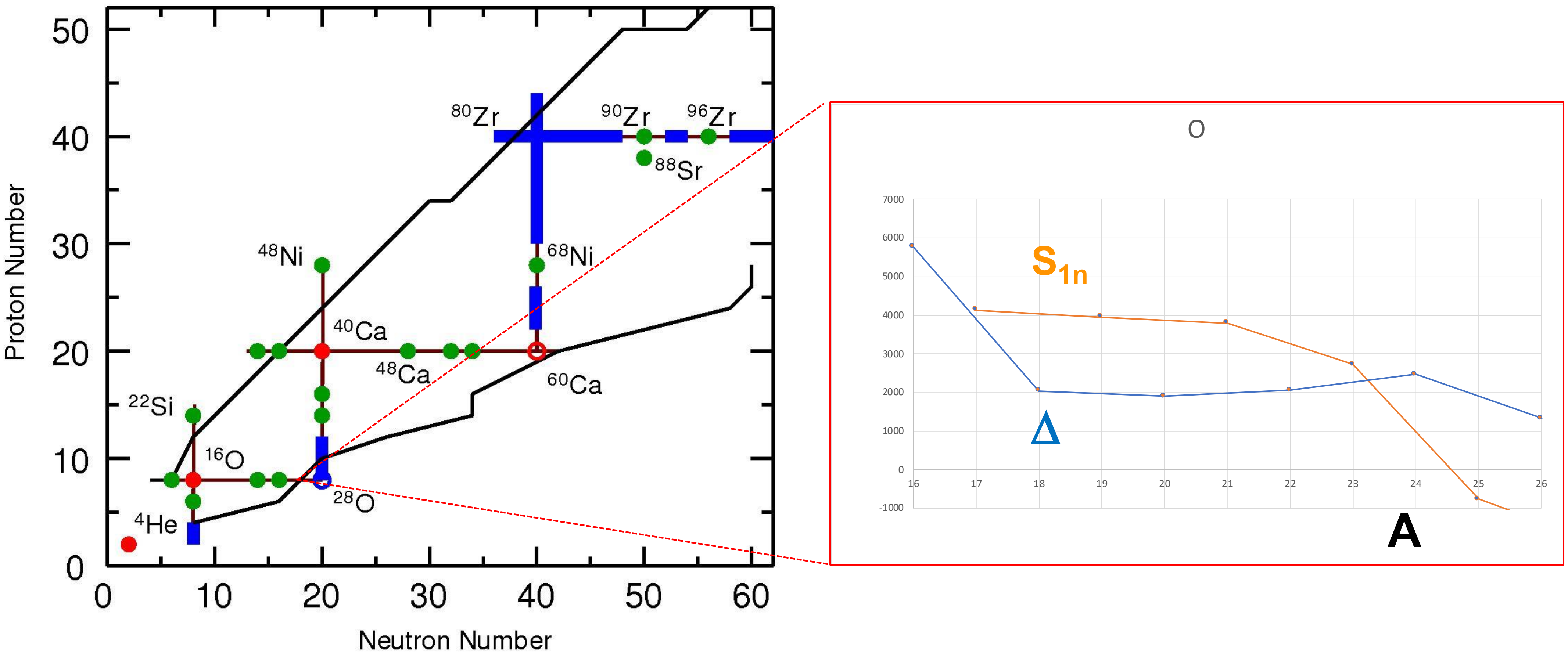


Superradiance and two-neutron transfer reactions





FRIB Experiments



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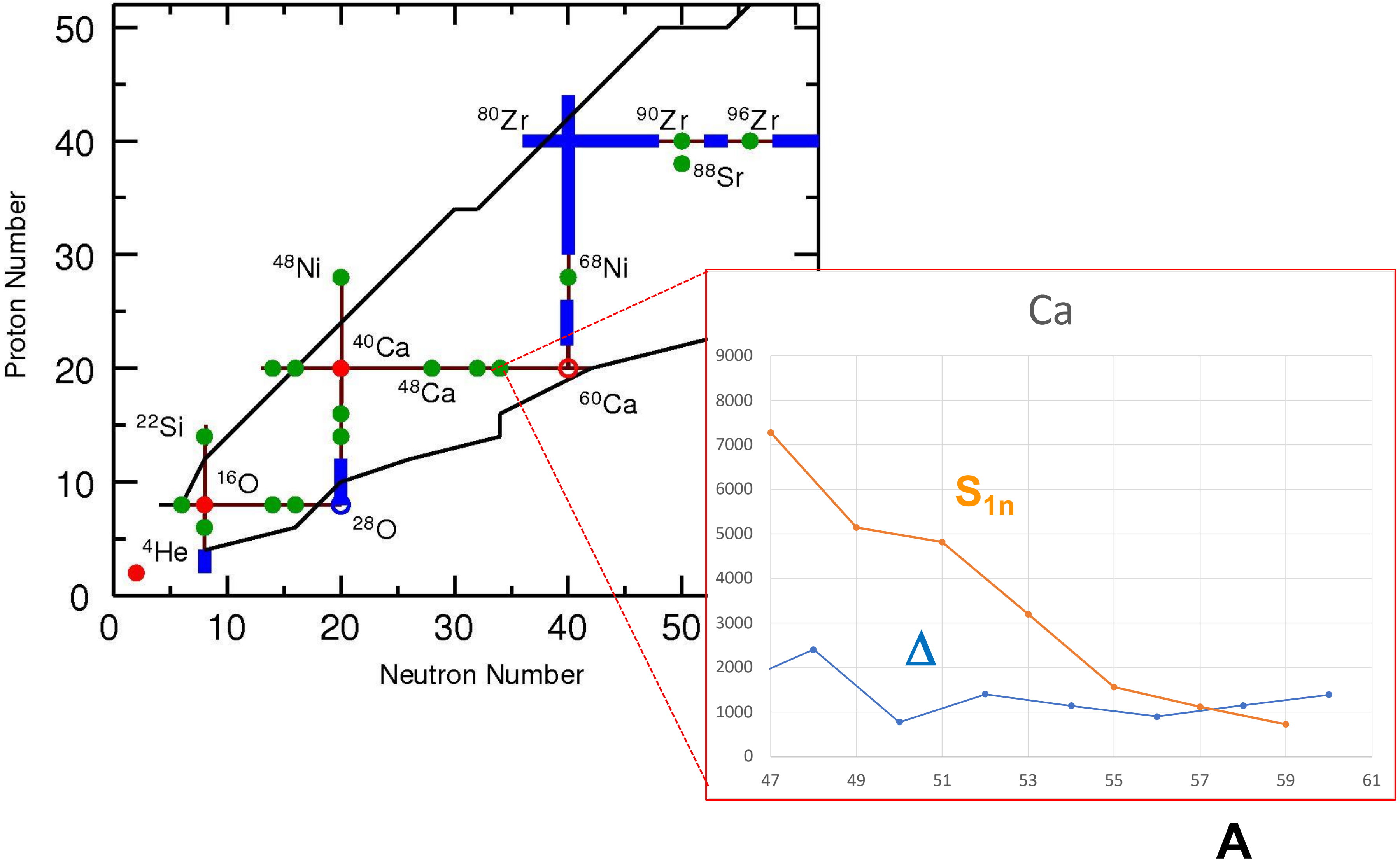

Summaries

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- ✓ Beta-decay Take 2 LBNL/ORNL
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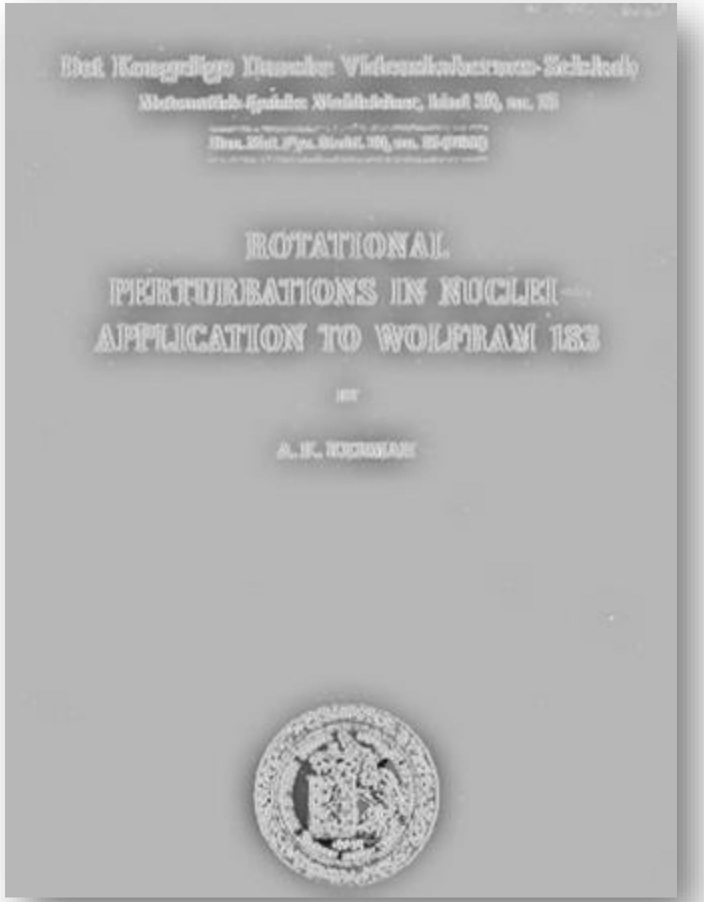
Motivation III : Kerman's Problem in the Continuum



Kerman's focus was on the mixing of the Nilsson orbits 1/2[510] and 3/2[512] in ^{183}W, ultimately achieving an excellent description of the perturbed energies. In this case, the PRM Hamiltonian is simply given by a 2x2 matrix

$$\begin{pmatrix} E_{3/2} & H_c \\ H_c & E_{1/2} \end{pmatrix} \rightarrow \quad H_c = -\frac{\hbar^2}{2\mathscr{I}}(I_+ j_- + I_- j_+)$$

Coriolis coupling

$$\varepsilon \left\{ \begin{array}{l} E_{3/2} = e_{3/2} + AI(I+1) \\ E_{1/2} = e_{1/2} + A(I(I+1) + (-)^{I+1/2} a(I+1/2)) \end{array} \right.$$

Rotational constant

$$A = \frac{\hbar^2}{2\mathscr{I}}$$

Decoupling parameter

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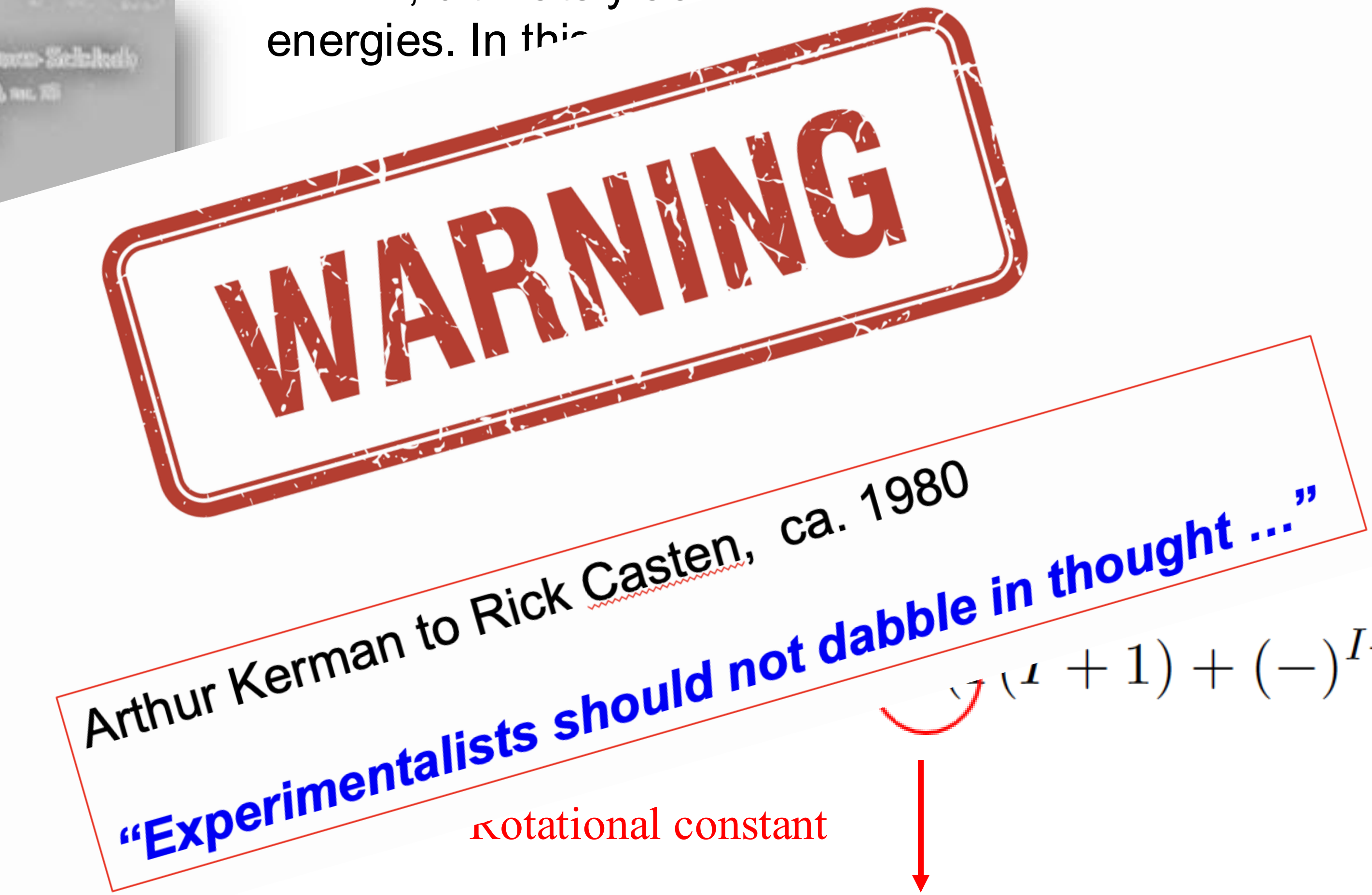

$$+ I_- j_+)$$

$$(I+1) + (-)^{I+1/2} a(I+1/2))$$

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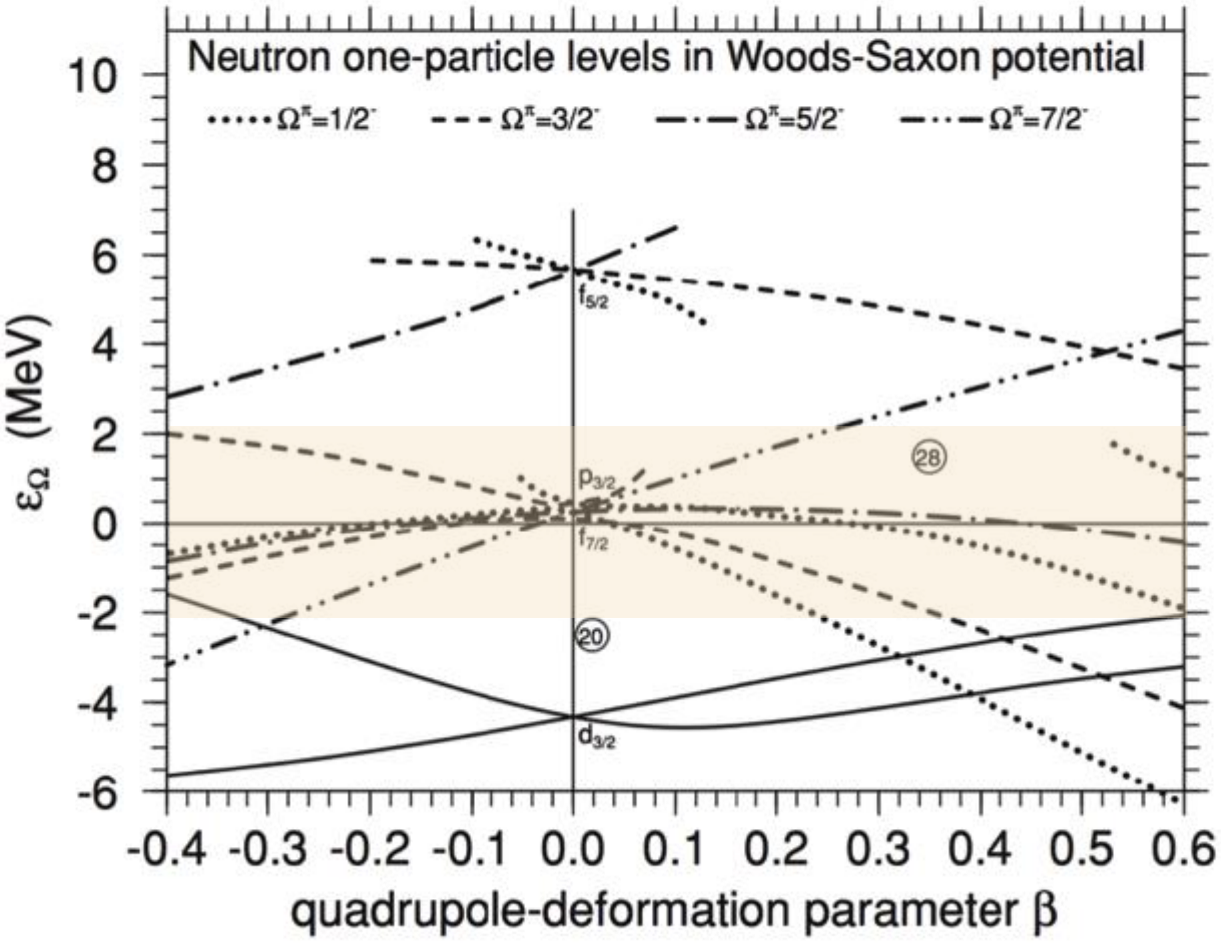
Weakly Bound Systems



^{39}Mg

I. Hamamoto, Phys. Rev. C 79, 014307 (2009)

K. Fossez, J. Rotureau, N. Michel, Quan Liu, and W. Nazarewicz, Phys. Rev. C 94, 054302 (2016)

A Toy Model: Strong Coupling limit $\Delta E_\Omega >> 2AIj$

ROTATIONAL BAND

CORIOLIS

$$\begin{pmatrix} AI^2 & -2AIj \\ -2AIj & AI^2 + \Delta E_\Omega - i\gamma \end{pmatrix}$$

NILSSON Energies

WIDTH

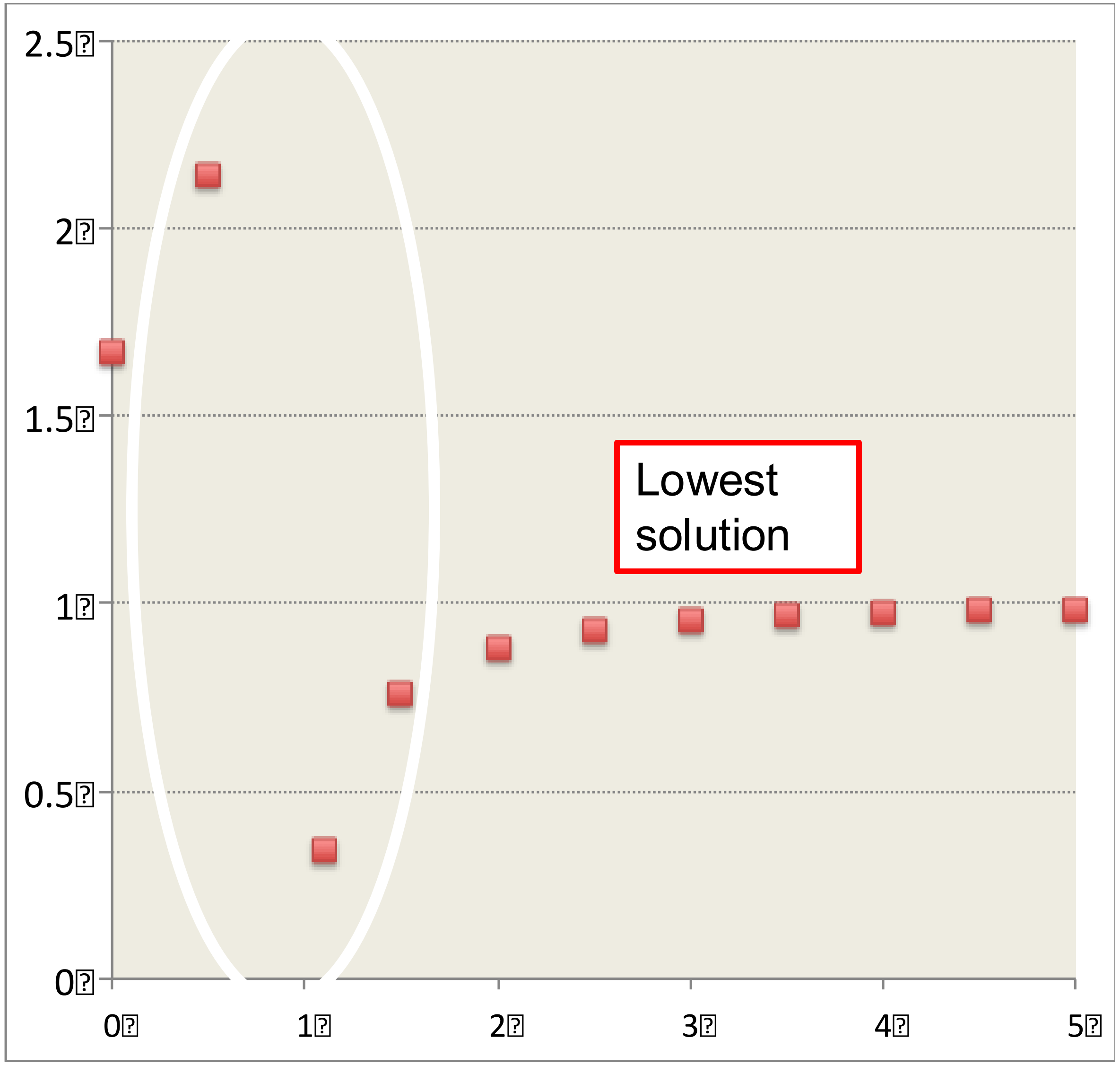
$$\frac{E_\pm}{A} = \Big(1 \pm \frac{4j^2 \Delta E_\Omega}{\Delta E_\Omega^2 - \gamma^2}\Big) I^2 + iW_\pm$$

RENORMALIZED

Dimensionless quantity

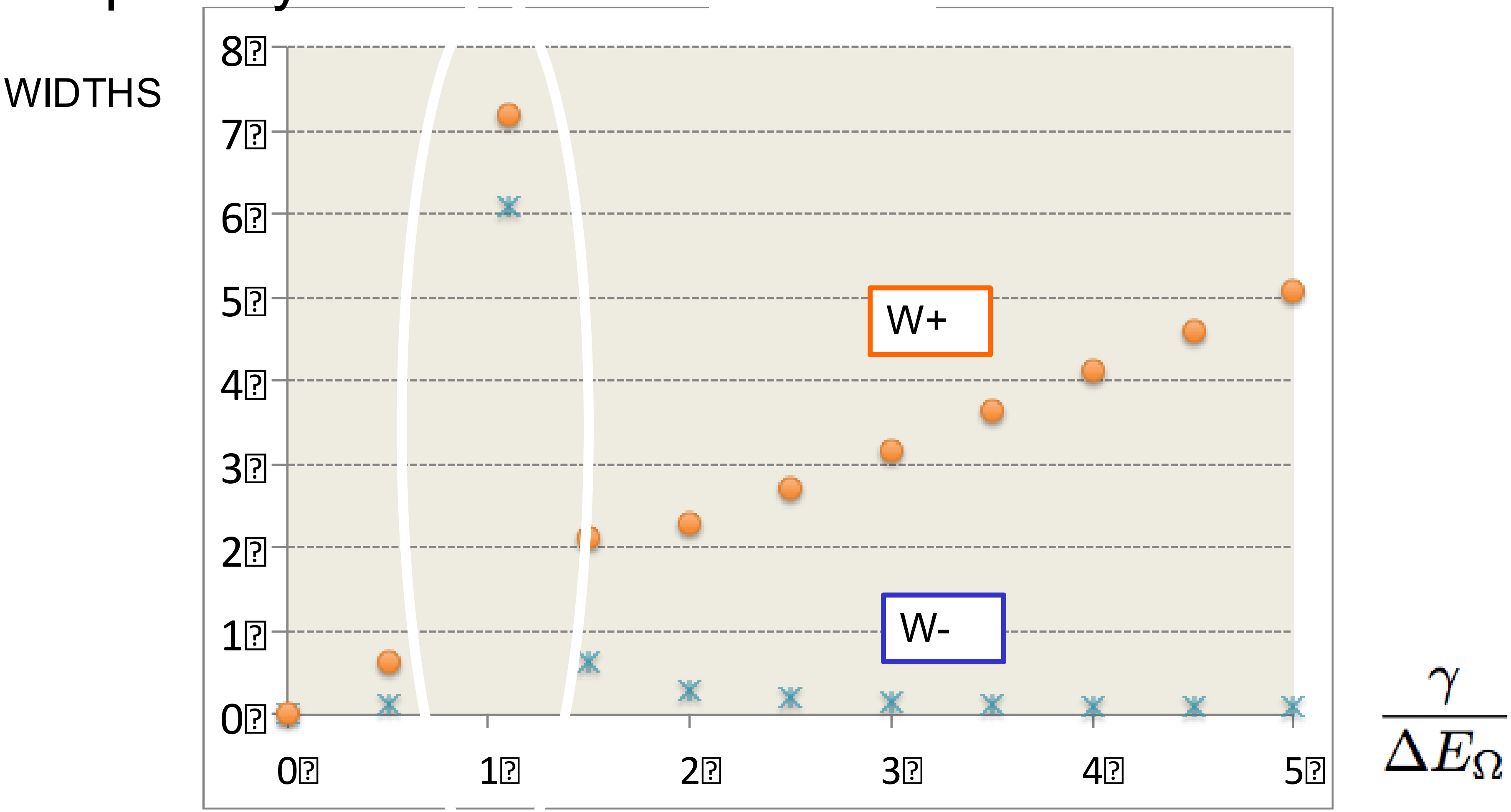
$$\frac{\gamma}{\Delta E_\Omega}$$

$\mathscr{I}^{(1)}$



$$\frac{\gamma}{\Delta E_\Omega}$$

Dimensionless quantity $\frac{\gamma}{\Delta E_\Omega}$



A Toy Model: decoupled limit
Degenerate Nilsson levels

ROTATIONAL BAND

CORIOLIS

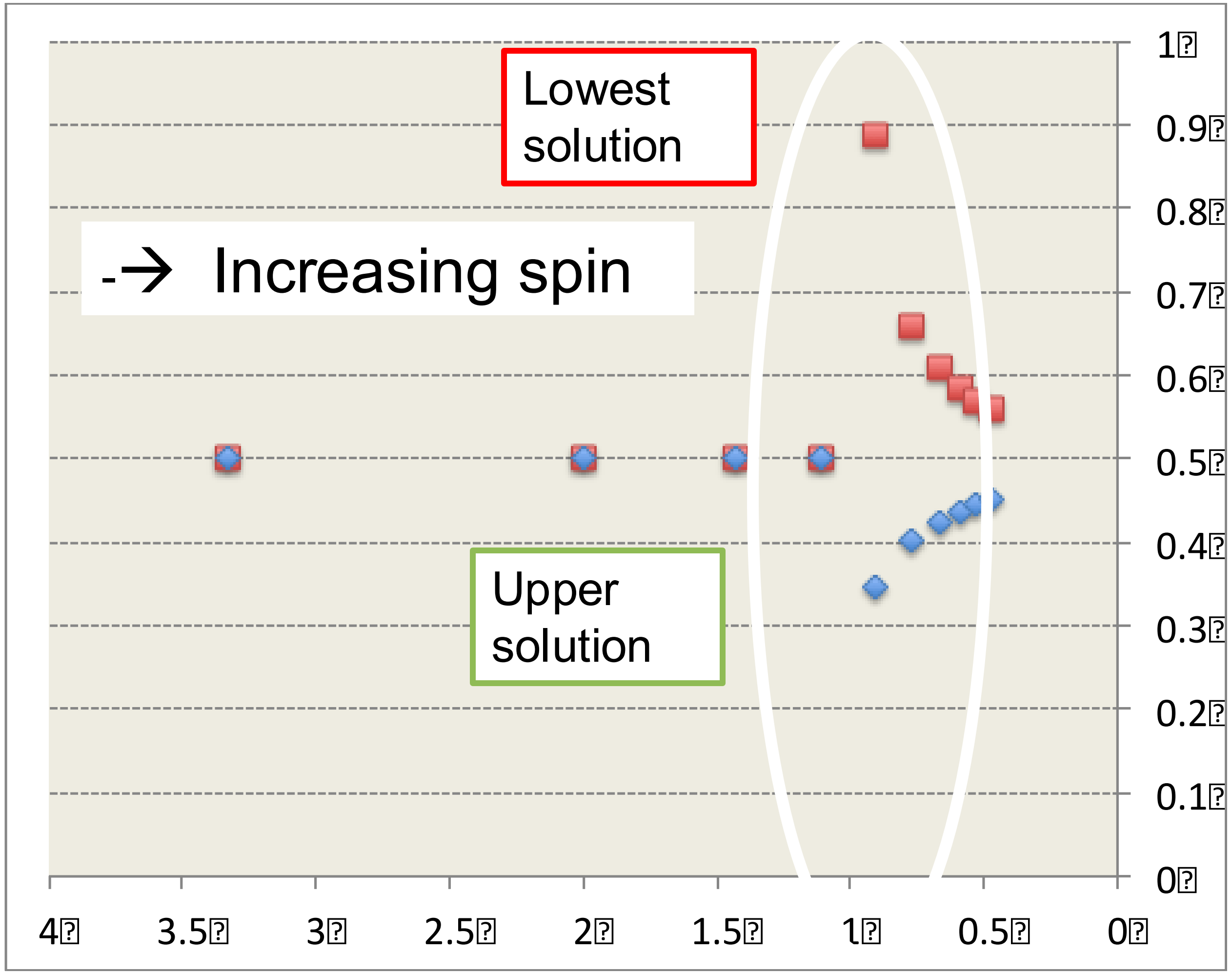
WIDTH

$$\begin{pmatrix} AI^2 & -2AIj \\ -2AIj & AI^2 - i\gamma \end{pmatrix}$$

$$\frac{E_\pm}{A} = I^2 \pm \sqrt{(2Ij)^2 - (\frac{\gamma}{2A})^2} - i\frac{\gamma}{2A}$$

In units of A

Dimensionless quantity $\frac{\gamma}{2Ij}$

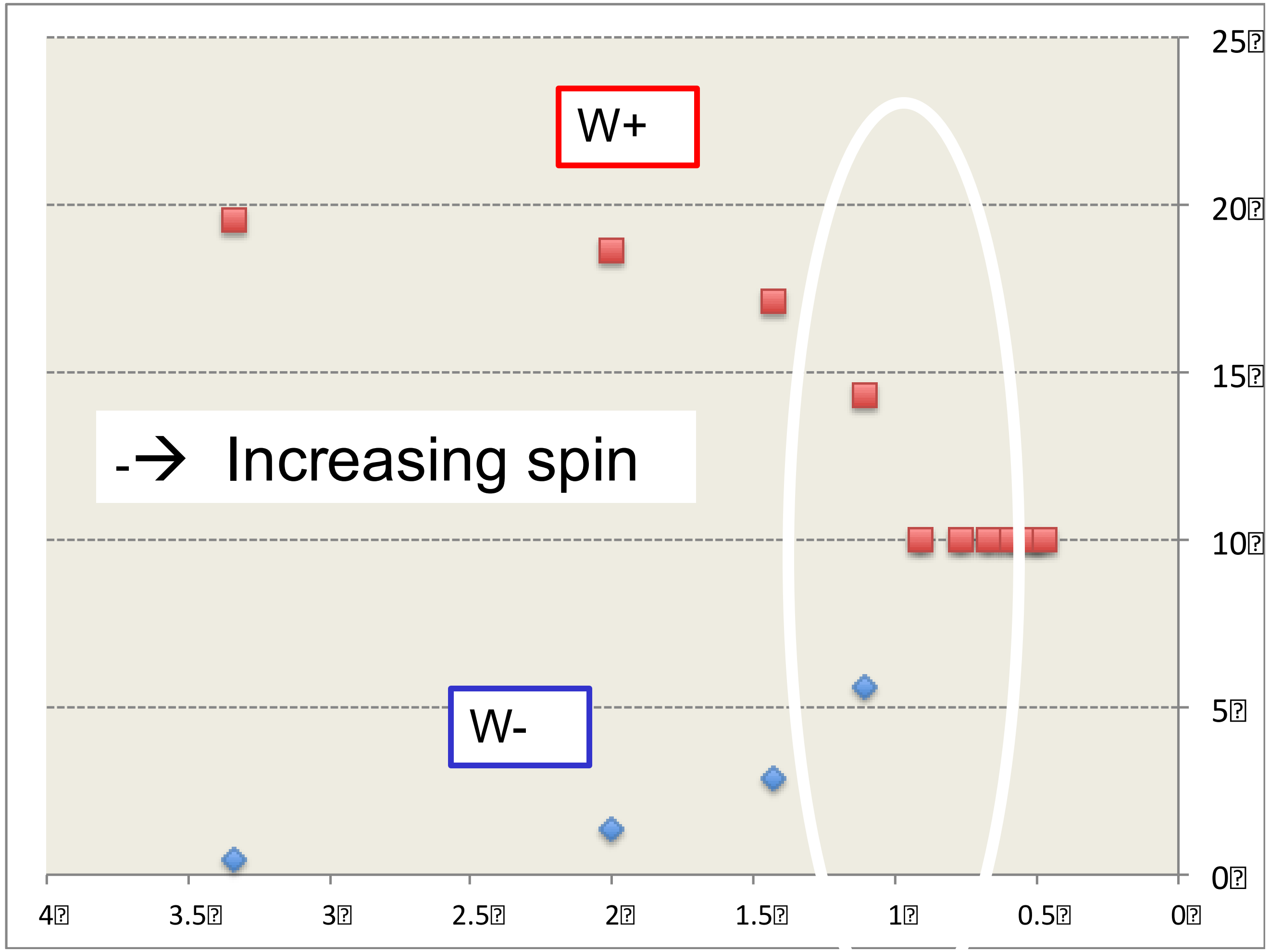


$\mathscr{I}^{(1)}$

$\frac{\gamma}{2Ij}$

OAK RIDGE National Laboratory

Dimensionless quantity $\frac{\gamma}{2Ij}$



WIDTHS

$\frac{\gamma}{2Ij}$

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LBNL Nuclear Structure Group

K. Wimmer, C. Hebborn, and G. Potel   (t,p)

R. Casten (Kerman)

Thank you Robert, for everything !

And Congratulations on your remarkable career and your outstanding accomplishments !

COMPADRE

Thank You !